

QUESTION 2 (25 MARKS)

Question 2 (a)

$$c_1 : x^2 + y^2 - 6x - 10y + 29 = 0$$

$$\text{Centre } P = (-g, -f) = (3, 5)$$

$$r_1 = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 25 - 29} = \sqrt{5}$$

$$c_2 : x^2 + y^2 - 2x - 2y - 43 = 0$$

$$\text{Centre } Q = (-g, -f) = (1, 1)$$

$$r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{1 + 1 + 43} = \sqrt{45} = 3\sqrt{5}$$

CENTRE AND RADIUS OF A CIRCLE

$$c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\text{Centre: } (-g, -f)$$

$$r = \sqrt{g^2 + f^2 - c}$$

Question 2 (b)

$$\text{Distance between the centres } |PQ| = \sqrt{(3-1)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$r_2 - r_1 = 3\sqrt{5} - \sqrt{5} = 2\sqrt{5}$$

Therefore, the circles touch internally.

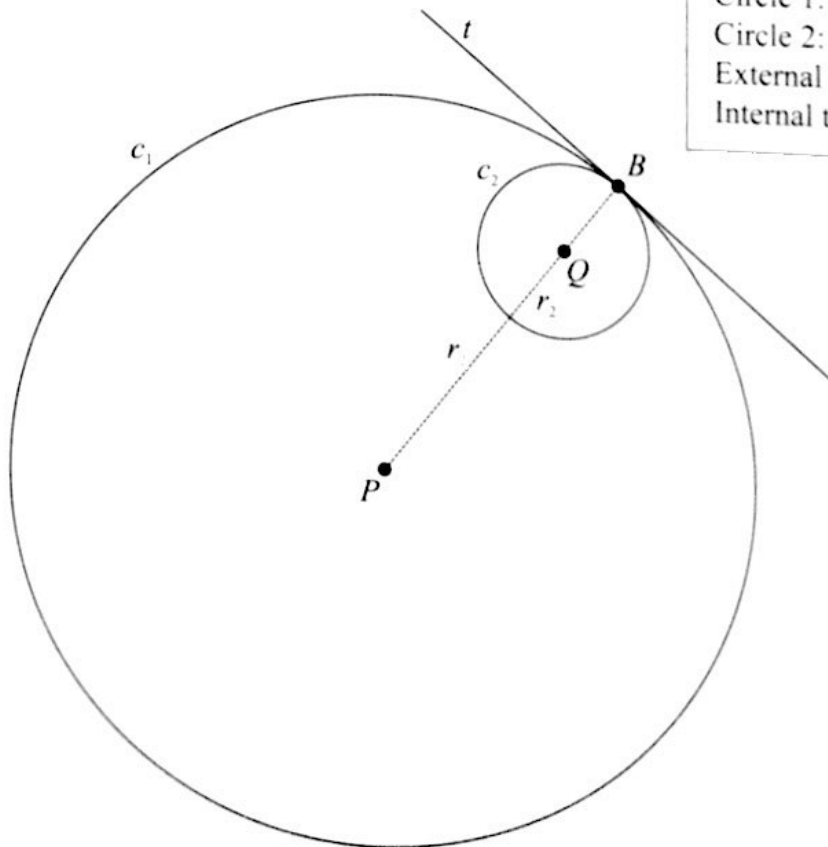
TOUCHING CIRCLES

Circle 1: Centre P , radius r_1

Circle 2: Centre Q , radius r_2

External touch: $|PQ| = r_1 + r_2$

Internal touch: $|PQ| = r_2 - r_1$



Question 2 (c)

Call this point B .

$$B(4, 7) \in c_1 ?$$

$$4^2 + 7^2 - 6(4) - 10(7) + 29 = 16 + 49 - 24 - 70 + 29 = 0$$

$$B(4, 7) \in c_2 ?$$

$$4^2 + 7^2 - 2(4) - 2(7) - 43 = 16 + 49 - 8 - 14 - 43 = 0$$

Therefore, $B(4, 7)$ is on each circle.

Question 2 (d)

$$P(3, 5), B(4, 7)$$

$$\text{Slope of } PB = \frac{7-5}{4-3} = 2 \quad [\text{Line } PB \text{ is perpendicular to the tangent } t.]$$

$$\text{Slope of } t = -\frac{1}{2}$$

$$\text{Equation of } t : m = -\frac{1}{2}, (x_1, y_1) = (4, 7)$$

$$y - 7 = -\frac{1}{2}(x - 4)$$

$$2y - 14 = -x + 4$$

$$x + 2y - 18 = 0$$