LC 2013 (Set C): Paper 1

Question 1 (25 marks)

Question 1 (a)

$\frac{4}{1+\sqrt{3}i}$ ← Multiply above and below by the conjugate of the denominator.

$= \frac{4}{(1+\sqrt{3}i)} \times \frac{(1-\sqrt{3}i)}{(1-\sqrt{3}i)}$

$= \frac{4(1-\sqrt{3}i)}{1-3i^2}$

$= \frac{4(1-\sqrt{3}i)}{4}$

$= 1-\sqrt{3}i$

Formula: Complex Numbers
Conjugates $\bar{z}$
$z = a+bi \Rightarrow \bar{z} = a-bi$
Multiplying a complex number by its conjugate:
$z\bar{z} = (a+bi)(a-bi) = a^2+b^2$

Marking Scheme Notes
Question 1 (a) [Scale 10C (0, 3, 7, 10)]
3: • Does not multiply by conjugate
• Drops i, or $i^2 \neq -1$
• Incomplete cross-multiplication
7: • Work not simplified

Question 1 (b)

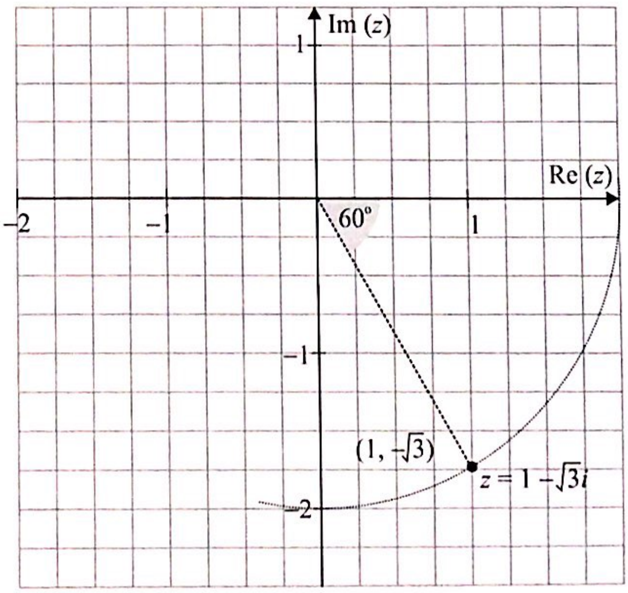
Plotting z: Find the modulus of z

$z = 1-\sqrt{3}i$

$|z| = \sqrt{1^2+(-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

Formula: Complex Numbers
Modulus $|z|$
$|z| = |a+bi| = \sqrt{a^2+b^2}$

Put the point of the compass at the origin and stretch the pencil out to 2 on the Real axis. Draw an arc as shown. The point $(1, -\sqrt{3})$ occurs at $x = 1$ ***or*** use your protractor to draw a 60° angle as shown.



Formula: Complex Numbers
$z = a+bi$ (Cartesian form)
$z = r(\cos\theta + i\sin\theta)$ (Polar form)

α is the reference angle in the first quadrant.

$|\tan\theta| = \left|\frac{-\sqrt{3}}{1}\right| = \sqrt{3} = \tan\alpha$

$\therefore \alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\therefore \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = 300°$

$\therefore z = 2\left\{\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right\}$

Marking Scheme Notes

Question 1 (b) [Scale 10C (0, 3, 7, 10)]

3:
- Work with α
- Work with θ
- Work with modulus
- Plotting z

7:
- Correct z but incorrect or no plotting

Note: Accept r, θ and plot for full marks.

Question 1 (c)

Formulae and Tables Book:
Algebra (page 20)
De Moivre's theorem
$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z = 2\left\{\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right\}$$

$$\therefore z^{10} = 2^{10}\left\{\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right\}^{10}$$

$$= 2^{10}\left\{\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right\}$$

$$= 2^{10}\left\{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right\}$$

$$= 2^{10}\left\{-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right\}$$

$$= -2^9(1 - \sqrt{3}i)$$

Marking Scheme Notes

Question 1 (c) [Scale 5C (0, 2, 4, 5)]

2:
- Some work with De Moivre
- De Moivre not used correctly

4:
- Answer not simplified
- n included in answer

Note: Allow for full marks candidates incorrect angle from (b), with correct conclusion.

0: no use of De Moivre.