Question 4 (25 marks)

Question 4 (a) (i)

Annual interest rate $r_A = 0{\cdot}4\% \Rightarrow i_A = 0{\cdot}04$
$(1 + i_M)^{12} = (1 + 0{\cdot}04)$, where i_M is the monthly interest rate

$$(1+i_M)^{12} = 1{\cdot}04$$

$$1+i_M = 1{\cdot}04^{\frac{1}{12}}$$

$$i_M = 1{\cdot}04^{\frac{1}{12}} - 1 = 0{\cdot}00327$$

$$\therefore r_M = 0{\cdot}327\%$$

> **Marking Scheme Notes**
> **Question 4 (a) (i) [Scale 5C* (0, 2, 4, 5)]**
> 2: • Any relevant first step
> • Correct statement with no work
> 4: • r not as a %
> **Note:** Incomplete rounding or incorrect rounding or no rounding gets 4 marks

Question 4 (a) (ii)

Call A the amount of money she saves each month. She saves for 36 months at a monthly interest rate of 0·327%. Her first saving amount A earns interest for 36 months, her second saving amount A earns monthly interest for 35 months and so on.

$$15\,000 = A\times1{\cdot}00327^{36} + A\times1{\cdot}00327^{35} + ... + A\times1{\cdot}00327^{2} + A\times1{\cdot}00327^{1}$$

$$15\,000 = A(1{\cdot}00327^{36} + 1{\cdot}00327^{35} + ... + 1{\cdot}00327^{2} + 1{\cdot}00327^{1})$$

$$15\,000 = A(1{\cdot}00327^{1} + 1{\cdot}00327^{2} + ... + 1{\cdot}00327^{35} + 1{\cdot}00327^{36})$$

This is a geometric series. You can work out the sum of these 36 savings.

$$a = 1{\cdot}00327,\ r = 1{\cdot}00327$$

$$S_{36} = \frac{1{\cdot}00327(1-1{\cdot}00327^{36})}{1-1{\cdot}00327}$$

$$1500 = A\left[\frac{1{\cdot}00327(1-1{\cdot}00327^{36})}{1-1{\cdot}00327}\right]$$

$$\therefore A = \frac{15\,000(1-1{\cdot}00327)}{1{\cdot}00327(1-1{\cdot}00327^{36})} = €392$$

> **Formulae and Tables Book**
> **Sequences and series:**
> **Geometric series** [page 22]
> $$S_n = \frac{a(1-r^n)}{1-r}$$

> **Marking Scheme Notes**
> **Question 4 (a) (ii) [Scale 10D* (0, 3, 5, 8, 10)]**
> 3: • Any relevant step
> • Reference to 1·00327
> 5: • Recognises G.P
> 8: • Expression for sum of G.P
> **Note:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks

Or

Question 4 (a) (ii)

Find the present value P of having a future value F of €15 000 in your account after 36 months.

$$P = \frac{F}{(1+i_M)^{36}}$$

$$= \frac{15\ 000}{1\cdot00327^{36}} = €13\ 336\cdot73$$

> **Formulae and Tables Book**
> **Financial mathematics: Compound interest** [page 30]
>
> $$F = P(1+i)^t$$
>
> t = Time period (in years)
> i = (Annual) rate of interest expressed as a decimal
> F = Final value
> P = Principal
> **Note:** The time period can be months or weeks instead of years provided the interest rate is given for that time period.

You can now ask the question in reverse: How much did I have to pay back every month to repay €13 336·73? Now I can use the amotisation formula.

> **Formulae and Tables Book**
> **Financial mathematics: Amortisation - mortgages and loans**
> (equal repayments at equal intervals) [page 31]
>
> $$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$
>
> t = Time period (in years)
> i = (Annual) rate of interest expressed as a decimal
> A = (Annual) repayment amount
> P = Principal
> **Note:** The time period can be months or weeks instead of years provided the interest rate is given for that time period.

$$A = 13\ 336\cdot73 \times \frac{0\cdot00327(1\cdot00327)^{36}}{(1\cdot00327)^{36} - 1} = €393\cdot30$$

> **Marking Scheme Notes**
> **Question 4 (a) (ii) [Scale 10D* (0, 3, 5, 8, 10)]**
> 3: • No present value or incorrect present value
> 5: • Present value correct
> 8: • Correct substitution of all values in formula
> **Note:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

Question 4 (b)

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= 15\ 000\left[\frac{0\cdot00866(1+0\cdot00866)^{36}}{(1+0\cdot00866)^{36} - 1}\right]$$

$$= 15\ 000\left[\frac{0.00866(1\cdot00866)^{36}}{(1\cdot00866)^{36} - 1}\right] = €487$$