

QUESTION 7 (50 MARKS)

Question 7 (a)

Ticket price: €20, attendance: 12 000

Ticket price: €19, attendance: 13 000

Ticket price: €18, attendance: 14 000

ANSWER: 14 000 people

Question 7 (b)

Call y the number in the stadium.

$$y = 12\,000 \text{ when } x = \text{€}20$$

$$y = 13\,000 \text{ when } x = \text{€}19$$

$$y = 14\,000 \text{ when } x = \text{€}18$$

$$y = 15\,000 \text{ when } x = \text{€}17$$

$$\begin{aligned} y &= 12\,000 + (20 - x)1000 \\ &= 12\,000 + 20\,000 - 1000x \\ &= 32\,000 - 1000x \end{aligned}$$

Question 7 (c)

$f(x) = \text{Number who attended} \times \text{Price}$

$$= y \times x$$

$$= (32\,000 - 1000x)x$$

$$= 32\,000x - 1000x^2$$

MARKING SCHEME NOTES

Question 7 (a) [Scale 10B (0, 5, 10)]

5: • $(20-18)1000$ or equivalent

Question 7 (b) [Scale 5B (0, 2, 5)]

2: • Expression $(20 - x)$

NOTE: Accept for 5 marks $12000 + (20 - x)1000$ or equivalent

Question 7 (c) [Scale 5B (0, 2, 5)]

2: • Correct number of people and/or correct rate in terms of x

Question 7 (d)

Differentiate the function with respect to x . Put the derivative equal to zero and solve for x to find the price that will give the maximum income.

$$f(x) = 32\,000x - 1000x^2$$

$$f'(x) = 32\,000 - 2000x$$

$$f'(x) = 0 \Rightarrow 32\,000 - 2000x = 0$$

$$32\,000 = 2000x$$

$$\therefore x = \frac{32\,000}{2000} = \text{€}16$$

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)

Put $\frac{dy}{dx} = 0$ and solve for x

MARKING SCHEME NOTES

Question 7 (d) [Scale 10B (0, 5, 10)]

- 5:
- Some correct differentiation of a quadratic function
 - $(32000 - 2000x) = 0$ or equivalent
 - Correct testing with incorrect deduction or no deduction
 - Possible to get full marks without use of calculus
 - Correct answer and no work

Question 7 (e)

$$x = \text{€}16$$

$$f(x) = 32\,000x - 1000x^2$$

$$f(16) = 32\,000(16) - 1000(16)^2 = \text{€}256\,000$$

Question 7 (f)

Find out the price of a ticket x that gives a full attendance of $y = 25\,000$.

$$y = 25\,000$$

$$y = 32\,000 - 1000x$$

$$25\,000 = 32\,000 - 1000x$$

$$1000x = 7000$$

$$\therefore x = \text{€}7$$

$$x = \text{€}7$$

$$f(x) = 32\,000x - 1000x^2$$

$$f(7) = 32\,000(7) - 1000(7)^2 = \text{€}175\,000$$

$$\text{Difference} = \text{€}256\,000 - \text{€}175\,000 = \text{€}81\,000$$

MARKING SCHEME NOTES

Question 7 (e) [Scale 5B (0, 2, 5)]

- 2:
- Some effort at substitution of 16 or equivalent

Question 7 (f) [Scale 10C (0, 3, 7, 10)]

- 3:
- Use of expression
 - Use of 25 000
 - Some use of tables
 - Equation solved
 - Price of ticket found
- 7:
- Total income from sales

Question 7 (g)

Call p the price of a family ticket.

If 1000 more family tickets are sold, then 4000 less single tickets will be sold. The loss overall will be €14 000.

$$\therefore 1000p - 4000 \times 16 = -14\ 000$$

$$1000p = 4000 \times 16 - 14\ 000$$

$$1000p = 50\ 000$$

$$\therefore p = \text{€}50$$

Let a = Number of single tickets

Let b = Number of family tickets

Attendance equation: $a + 4b = 25\ 000$...(1)

Income equation: $16a + 50b = 365\ 000$...(2)

Solve equations (1) and (2):

$$a + 4b = 25\ 000 \dots\dots\dots(1)(\times 16)$$

$$16a + 50b = 365\ 000 \dots\dots(2)(\times -1)$$

$$16a + 64b = 400\ 000$$

$$-16a - 50b = -365\ 000$$

$$14b = 35\ 000 \Rightarrow b = \frac{35\ 000}{14} = 2500$$

ANSWER: 2500 family tickets

MARKING SCHEME NOTES

Question 7 (g) [Scale 5D (0, 2, 3, 4, 5)]

- 2: • Correct number of family tickets
- One equation only
- Income from single tickets
- Income from family tickets
- 3: • Two correct linear equations for income in two unknowns
- $y = 50$ without work, or $p = 15\ 000$ without work
- 4: • Correct value for p (single ticket)
- Correct value for y (family ticket)

OR

Question 7 (g) [Scale 5D (0, 2, 3, 4, 5)]

- 2: • One correct equation
- 3: • Three correct equations
- 4: • One unknown calculated