Question 8 (50 marks)

Question 8 (a)

$s(t) = 6t + 0\cdot3t^2 - 0\cdot01t^3$ ← Replace t by 10 in $s(t)$.

$$s(10) = 6(10) + 0\cdot3(10)^2 - 0\cdot01(10)^3$$
$$= 60 + 30 - 10$$
$$= 80 \text{ m}$$

Marking Scheme Notes
Question 8 (a) [Scale 10C* (0, 3, 7, 10)]
3: • Some correct substitution into $s(t)$
7: • Correct substitution.
• $10s = 80 \Rightarrow s = 8$
Full Marks: Correct answer without work

Question 8 (b)

$s = 6t + 0\cdot3t^2 - 0\cdot01t^3$

$v = \dfrac{ds}{dt} = 6 + 0\cdot6t - 0\cdot03t^2$

$v = 8\cdot25 \text{ m s}^{-1}$: $8\cdot25 = 6 + 0\cdot6t - 0\cdot03t^2$

$0\cdot03t^2 - 0\cdot6t + 2\cdot25 = 0$

$t^2 - 20t + 75 = 0$

$(t-5)(t-15) = 0$

$\therefore t = 5 \text{ s}, 15 \text{ s}$

Answer: $t = 5$ s

Note: Ignore the other solution as the function used is specified for the first 10 s.

Formulae: Speed
Velocity $v = \dfrac{\text{Change } (d) \text{ in distance } s}{\text{Change } (d) \text{ in time } t}$
$v = \dfrac{ds}{dt}$

Formulae and Tables Book
Calculus: Derivatives [page 25]
$y = x^n \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$

Marking Scheme Notes
Question 8 (b) [Scale 10D* (0, 3, 5, 8, 10)]
3: • Any correct differentiation
5: • $\dfrac{ds}{dt}$ found correctly
8: • Fails to select correct value for t

Question 8 (c)

$v = 6 + 0\cdot6t - 0\cdot03t^2$

$a = \dfrac{dv}{dt} = 0\cdot6 - 0\cdot06t$

$a = 0\cdot006 \text{ m s}^{-2}$: $0\cdot006 = 0\cdot6 - 0\cdot06t$

$0\cdot06t = 0\cdot6 - 0\cdot006$

$t = \dfrac{0\cdot6 - 0\cdot006}{0\cdot06} = 9\cdot9 \text{ s}$

Formulae: Acceleration
Acceleration $a = \dfrac{\text{Change } (d) \text{ in velocity } v}{\text{Change } (d) \text{ in time } t}$
$a = \dfrac{dv}{dt}$

Marking Scheme Notes
Question 8 (c) [Scale 10C* (0, 3, 7, 10)]
3: • Some correct differentiation of the first derivative
7: • Sets up correct equation.

Question 8 (d)

You discovered in **part (a)** that the drop falls 80 m in the first 10 s. Therefore, it falls the remaining 540 m at constant velocity. Calculate this velocity.

$t = 10$ s: $v = 6 + 0 \cdot 6t - 0 \cdot 03t^2$

$v = 6 + 0 \cdot 6(10) - 0 \cdot 03(10)^2 = 9 \text{ m s}^{-1}$

$$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{540}{9} = 60 \text{ s}$$

Answer: Total time $= 10 + 60 = 70$ s

Marking Scheme Notes
Question 8 (d) [Scale 10D* (0, 3, 5, 8, 10)]
3: • Some correct substitution
• Finds distance travelled after initial 10 seconds i.e. 540m
• Correct differentiation (relevant for terminal velocity)
5: • Finds k
8: • Finds time travelling at terminal velocity and stops

Question 8 (e)

$$\frac{dV}{dt} = 6 \text{ mm s}^{-1}$$

$$\left(\frac{dr}{dt}\right)_{r=1\cdot5} = ?$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$$

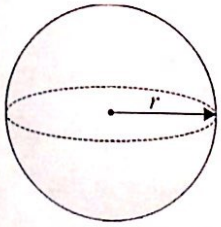
$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 6 = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{3}{2\pi r^2}$$

$$\therefore \left(\frac{dr}{dt}\right)_{r=1\cdot5} = \frac{3}{2\pi(1\cdot5)^2}$$

$$= 0 \cdot 2122 \text{ mm s}^{-1}$$

Formulae and Tables Book
Surface area and volume: Sphere [page 8]



$A = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Marking Scheme Notes
Question 8 (e) [Scale 10D* (0, 3, 5, 8, 10)]
3: • $V = \frac{4}{3}\pi r^3$
• Any relevant step
5: • Any two of $\frac{dv}{dt} = 6$, $\frac{dv}{dr} = 4\pi r^2$, $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$
8: • Correct substitution into chain rule
• Fails to substitute for r