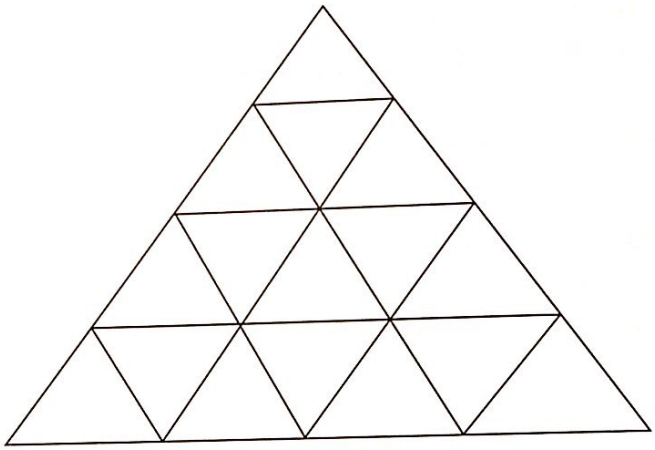
Question 9 (50 marks)

Question 9 (a) (i)



Question 9 (a) (ii)

Pattern	1st	2nd	3rd	4th
Number of small triangles	1	4	9	16
Number of matchsticks	3	9	18	30

Marking Scheme Notes

Question 9 (a) (i) [Scale 5B (0, 2, 5)]

2: • Incomplete 4th line

Question 9 (a) (ii) [Scale 5C (0, 2, 4, 5)]

2: • One or two correct entries in empty boxes

4: • Three correct entries in empty boxes

Question 9 (b)

Number of triangles in the 1st pattern ($n = 1$): $T_1 = 1 = 1^2$

Number of triangles in the 2nd pattern ($n = 2$): $T_2 = 4 = 2^2$

Number of triangles in the 3rd pattern ($n = 3$): $T_3 = 9 = 3^2$

Number of triangles in the 4th pattern ($n = 4$): $T_4 = 16 = 4^2$

⋮

Number of triangles in the n^{th} pattern ($n = n$): $T_n = n^2$

OR

Terms: 1, 4, 9, 16, 25

Sequence of **first** differences: 3, 5, 7, 9

Sequence of **second** differences: 2, 2, 2

A constant term of 2 in the sequence of second differences implies the number of triangles in each pattern forms a quadratic sequence.

Marking Scheme Notes
Question 9 (b) [Scale 10B (0, 5, 10)]
5: • $1^2, 2^2, 3^2$ etc – recognising the natural numbers squared
or
Question 9 (b) [Scale 10B (0, 5, 10)]
5: • Second differences calculated

Question 9 (c)

Number of matchsticks to turn 1st pattern into 2nd pattern ($n = 2$): $6 = 3(2)$

Number of matchsticks to turn 2nd pattern into 3rd pattern ($n = 3$): $9 = 3(3)$

Number of matchsticks to turn 3rd pattern into 4th pattern ($n = 4$): $12 = 3(4)$

⋮

Number of matchsticks to turn $(n-1)^{th}$ pattern into n^{th} pattern ($n = n$): $3n$

Marking Scheme Notes
Question 9 (c) [Scale 10B (0, 5, 10)]
5: • Recognition of series 6, 9, 12,.... or similar
or
Question 9 (c) [Scale 10B (0, 5, 10)]
5: • Second differences calculated

Question 9 (d)

$u_n = an^2 + bn$

$u_1 = a(1)^2 + b(1) = 3 \Rightarrow a + b = 3 ...(1)$

$u_2 = a(2)^2 + b(2) = 9 \Rightarrow 4a + 2b = 9 ...(2)$

$a + b = 3 .....(1)(\times -2)$

$4a + 2b = 9 ...(2)$

$-2a - 2b = -6$

$4a + 2b = 9$

$2a \quad = 3 \Rightarrow a = \frac{3}{2}$

$a = \frac{3}{2} : a + b = 3 \Rightarrow b = 3 - \frac{3}{2} = \frac{3}{2}$

Answer: $a = \frac{3}{2}, b = \frac{3}{2}$

Marking Scheme Notes
Question 9 (d) [Scale 10C (0, 3, 7, 10)]
3: • One linear equation in a and b, e.g. u_1: $a + b = 3$
7: • Two correct linear equations
or
Question 9 (d) [Scale 10C (0, 3, 7, 10)]
3: • Recognition of A.P.
7: • $a = d = 3$ and some use of S_n formula

Question 9 (e)

$u_n = an^2 + bn = 4134$

$\frac{3}{2}n^2 + \frac{3}{2}n = 4134$

$3n^2 + 3n - 8268 = 0$

$n^2 + n - 2756 = 0$

$(n-52)(n+53) = 0$

$\therefore n = 52$

Number of triangles $= n^2 = 52^2 = 2704$

Marking Scheme Notes

Question 9 (e) [Scale 10C (0, 3, 7, 10)]

3: • Expression of u_n in one variable only
• Quadratic equation

7: • Values of n