Question 2 May 17 **(25 marks)**

Let $z_1 = 1-2i$, where $i^2 = -1$.

(a) The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15 = 0$. Find the other two roots of the equation.

All real coefficients $\Rightarrow$ $1-2i$ and $\boxed{1+2i}$

$2z^3 - 7z^2 + 16z - 15 = 0$

$\therefore\ z^3 - \frac{7}{2}z^2 + 8z - \frac{15}{2} = 0$

But $z^3 - Sz^2 + SPPz - P = 0$

$\therefore\ \frac{7}{2} = \text{sum of roots} = 1 - \cancel{2i} + 1 + \cancel{2i} + z_3$

$\therefore\ \frac{7}{2} = 2 + z_3 \Rightarrow \boxed{z_3 = \frac{3}{2}}$

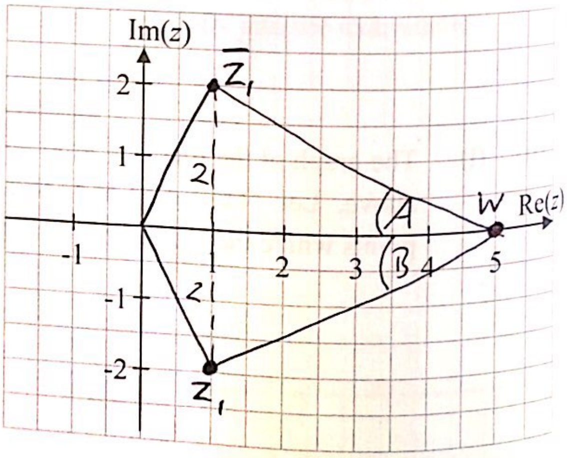
(b) **(i)** Let $w = z_1\overline{z}_1$, where $\overline{z}_1$ is the conjugate of z_1. Plot z_1, $\overline{z}_1$ and w on the Argand diagram and label each point.

$w = (1-2i)(1+2i)$

$= 1 - 4i^2 \quad i^2 = -1$

$= 1 + 4$

$= 5$



(ii) Find the measure of the acute angle, $\overline{z}_1 w z_1$, formed by joining $\overline{z}_1$ to w to z_1 on the diagram above. Give your answer correct to the nearest degree.

$\tan A = \frac{2}{4} = \frac{1}{2} \Rightarrow A = 26.6^\circ$

$\tan B = \frac{2}{4} = \frac{1}{2} \Rightarrow B = 26.6^\circ$

$\therefore\ |\angle \overline{z}_1 w z_1| = A + B \approx \boxed{53^\circ}$