Marking Scheme Notes
Question 4 (b) [Scale 10D (0, 3, 7, 8, 10)]
3: • Either $\frac{du}{dx}$ or $\frac{dv}{dx}$ correct
• No differentiation but writes $f'(x)=\frac{1}{4}$
7: • $f'(x)$ correct but not simplified
8: • Correct values of x from students work

Question 5 (25 marks)

Question 5 (a)

$$\int 5\cos 3x\,dx$$
$$=5\int \cos 3x\,dx$$
$$=\tfrac{5}{3}\sin 3x+c$$

Formulae and Tables Book
Calculus: Integrals [page 26]
$$\int \cos x\,dx=\sin x+c$$
$$\int \cos(ax+b)\,dx=\tfrac{1}{a}\sin(ax+b)+c$$

Marking Scheme Notes
Question 5 (a) [Scale 5B (0, 3, 5)]
3: • Some correct integration
• Integrand does not contain c
• c only

Question 5 (b) (i)

$$\frac{dy}{dx}=2x-2$$
$$\int dy=\int (2x-2)\,dx$$
$$y=\frac{2x^2}{2}-2x+c$$
$$=x^2-2x+c$$

Formulae and Tables Book
Calculus: Integrals [page 26]
$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+c,\ n\neq -1$$

(−2, 0) is on the curve.

$$y=x^2-2x+c$$
$$x=-2,\ y=0\Rightarrow 0=(-2)^2-2(-2)+c$$
$$0=4+4+c$$
$$\therefore c=-8$$
$$\therefore y=f(x)=x^2-2x-8$$

Marking Scheme Notes
Question 5 (b) (i) [Scale 10C (0, 5, 7, 10)]
5: • Some correct integration
• Integrand does not contain c
• c only
• $\frac{dy}{dx}=2x-2$ or $\frac{dy}{dx}=$ slope of tangent
7: • Substitutes (−2, 0) but c not simplified
Note: must have 'c' in equation to get high partial marks

Question 5 (b) (ii)

$$\bar{f} = \frac{1}{3-0}\int_0^3 (x^2 - 2x - 8)\,dx$$

$$= \frac{1}{3}\left[\frac{x^3}{3} - \frac{2x^2}{2} - 8x\right]_0^3 = \frac{1}{3}\left[\frac{x^3}{3} - x^2 - 8x\right]_0^3$$

$$= \frac{1}{3}\left[\left(\frac{3^3}{3} - 3^2 - 8(3)\right) - 0\right]$$

$$= \tfrac{1}{3}[9 - 9 - 24] = -8$$

Formula
Average value of f

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Formulae and Tables Book
Calculus: Integrals [page 26]

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$$

Marking Scheme Notes
Question 5 (b) (ii) [Scale 10C (0, 5, 7, 10)]

5:
- Correct formula only
- Some correct integration
- Indication of integration with correct limits
- If only values used e.g. $f(0), f(1), f(2)$ etc. when $0 \le x \le 3$, give Low Partial Credit for two or more values

7:
- Limits inserted into function but not calculated
- $\frac{1}{(b-a)}$ missing from formula

Note: NO CREDIT – differentiation
NO CREDIT – no integration

Question 6 (25 marks)

Question 6 (a) (i)

$T_n = \ln a^n = n \ln a$

$T_1 = \ln a,\ T_2 = 2\ln a,\ T_3 = 3\ln a$

$T_3 - T_2 = 3\ln a - 2\ln a = \ln a$

$T_2 - T_1 = 2\ln a - \ln a = \ln a$

$\therefore T_3 - T_2 = T_2 - T_1$

Therefore, T_1, T_2 and T_3 are in arithmetic sequence.

Formulae and Tables Book
Indices and logs [page 21]

$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a(x^q) = q\log_a x$

$\log_a 1 = 0$

$\log_a\left(\frac{1}{x}\right) = -\log_a x$

Question 6 (a) (ii)

$T_n = \ln a^n = n\ln a$

$T_{n+1} = (n+1)\ln a$

$T_{n+1} - T_n = (n+1)\ln a - n\ln a$

$= n\ln a + \ln a - n\ln a$

$= \ln a = \text{Constant } (d)$

Therefore, the sequence is arithmetic with common difference $d = \ln a$.