

**QUESTION 6 (25 MARKS)**

**Question 6 (a) (i)**

$$T_n = \ln a^n = n \ln a$$

$$T_1 = \ln a, T_2 = 2 \ln a, T_3 = 3 \ln a$$

$$T_3 - T_2 = 3 \ln a - 2 \ln a = \ln a$$

$$T_2 - T_1 = 2 \ln a - \ln a = \ln a$$

$$\therefore T_3 - T_2 = T_2 - T_1$$

Therefore,  $T_1, T_2$  and  $T_3$  are in arithmetic sequence.

**Question 6 (a) (ii)**

$$T_n = \ln a^n = n \ln a$$

$$T_{n+1} = (n+1) \ln a$$

$$\begin{aligned} T_{n+1} - T_n &= (n+1) \ln a - n \ln a \\ &= n \ln a + \ln a - n \ln a \\ &= \ln a = \text{Constant } (d) \end{aligned}$$

Therefore, the sequence is arithmetic with common difference  $d = \ln a$ .

**FORMULAE AND TABLES BOOK**  
**Indices and logs [page 21]**

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

**MARKING SCHEME NOTES**

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

**Question 6 (a) (i) [Scale 10C (0, 5, 7, 10)]**

- 5: • Only one term correct
- 7: • Either  $(T_2 - T_1)$  or  $(T_3 - T_2)$  correct

**Question 6 (a) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • Uses two consecutive general terms  
• Recognition of common difference and no more
- 4: • Shows series arithmetic but does not state common difference

**Question 6 (b)**

$$T_1 + T_2 + T_3 + \dots + T_{98} + T_{99} + T_{100} = S_{100} = 10\,100$$

$$T_1 = \ln a, d = \ln a, n = 100$$

$$S_{100} = \frac{100}{2}[2 \ln a + 99 \ln a] = 10\,100$$

$$50[101 \ln a] = 10\,100$$

$$\ln a = 2$$

$$\therefore a = e^2$$

**FORMULAE AND TABLES BOOK**  
**Sequences and series: Arithmetic series**

[page 22]

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$a$  is the first term

$d$  is the common difference

**MARKING SCHEME NOTES**

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

**Question 6 (b) [Scale 5C (0, 3, 4, 5)]**

- 3: • Writes three or more terms in form of  $n$  and  $\ln a$   
• Correct AP formula stated  
• Correct  $T_n$  formula
- 4: • Correct substitution into formula  
•  $\ln a = 2$  and does not finish

NOTE: accept  $a = e^2$  for full marks

**Question 6 (c)**

**LHS**

$$\begin{aligned} & T_1 + T_2 + T_3 + \dots + T_{10} + 100d \\ &= S_{10} + 100d \\ &= \frac{10}{2}[2 \ln a + 9 \ln a] + 100 \ln a \\ &= 5[11 \ln a] + 100 \ln a \\ &= 155 \ln a \end{aligned}$$

**RHS**

$$\begin{aligned} & T_{11} + T_{12} + T_{13} + \dots + T_{20} \\ &= S_{20} - S_{10} \\ &= \frac{20}{2}[2 \ln a + 19 \ln a] - \frac{10}{2}[2 \ln a + 9 \ln a] \\ &= 10[21 \ln a] - 5[11 \ln a] \\ &= 210 \ln a - 55 \ln a \\ &= 155 \ln a \end{aligned}$$