Question 9 (a)

$y = 100 - 23 = 77^{\circ}\,\text{C}$

$t = 0,\ y = 77$

$y = Ae^{kt}$

$77 = Ae^{k(0)}$

$77 = Ae^{0}$

$\therefore A = 77$

Question 9 (b)

$t = 5,\ y = 88 - 23 = 65$

$y = Ae^{kt}$

$65 = 77e^{5k}$

$e^{5k} = \dfrac{65}{77} \Rightarrow 5k = \ln\left(\dfrac{65}{77}\right)$

$k = \dfrac{1}{5}\ln\left(\dfrac{65}{77}\right) = -0\cdot0339$

Marking Scheme Notes
Question 9 (a) [Scale 10C (0, 5, 7, 10)]
5: • Value of y only
   • Some use of 100 and/or 23
7: • Correct substitution into equation
   • A calculated from incorrect y

Question 9 (b) [Scale 10C (0, 5, 7, 10)]
5: • Value of y only
   • Some use of 88 and/or 23
7: • Correct expression for k
   • k calculated from incorrect y

Question 9 (c)

$t = ?$, $y = 50 - 23 = 27$

$y = Ae^{kt}$

$27 = 77e^{-0.0339t}$

$e^{-0.0339t} = \dfrac{27}{77} \Rightarrow -0.0339t = \ln\left(\dfrac{27}{77}\right)$

$\therefore t = -\dfrac{1}{0.0339}\ln\left(\dfrac{27}{77}\right) = 30.91 \approx 31$ minutes

Marking Scheme Notes
Question 9 (c) [Scale 10C (0, 5, 7, 10)]
5: • Value of y only
7: • Correct expression for t
   • t calculated from incorrect y

Question 9 (d)

$y = f(t) = 77e^{-0.0339t}$

$t = 0: f(0) = 77e^{-0.0339(0)} = 77$

$t = 10: f(0) = 77e^{-0.0339(10)} = 54.9 \approx 55$

$t = 20: f(0) = 77e^{-0.0339(20)} = 39.1 \approx 39$

$t = 30: f(0) = 77e^{-0.0339(30)} = 27.8 \approx 28$

$t = 40: f(0) = 77e^{-0.0339(40)} = 19.8 \approx 20$

$t = 50: f(0) = 77e^{-0.0339(50)} = 14.1 \approx 14$
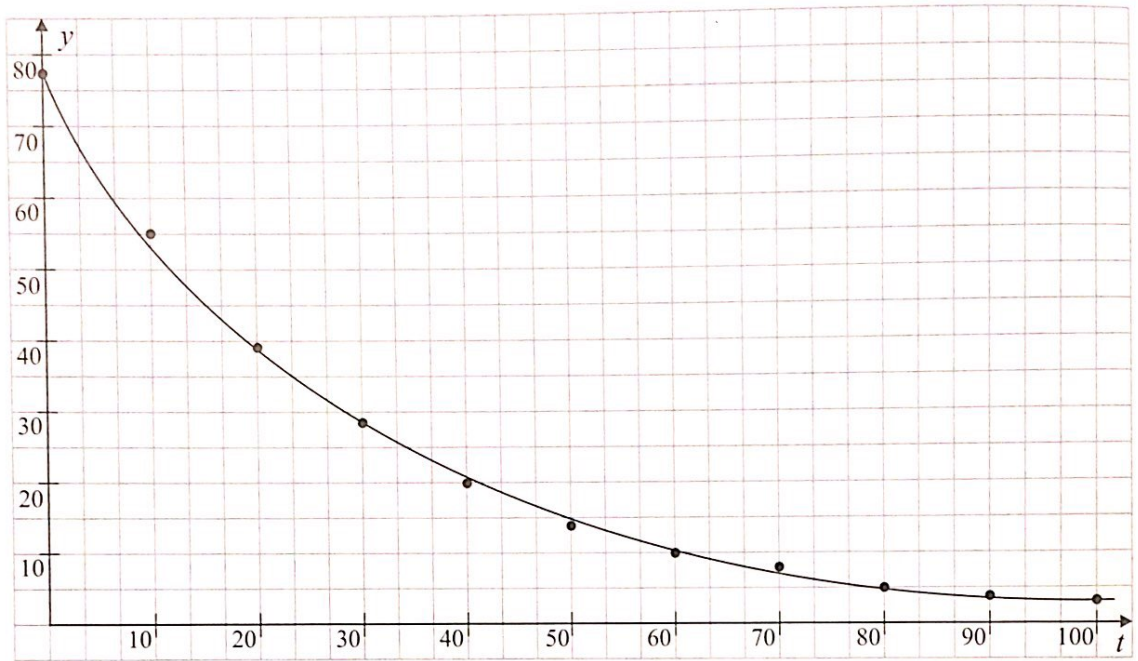
$t = 60: f(0) = 77e^{-0.0339(60)} = 10.1 \approx 10$

$t = 70: f(0) = 77e^{-0.0339(70)} = 7.2 \approx 7$

$t = 80: f(0) = 77e^{-0.0339(80)} = 5.1 \approx 5$

$t = 90: f(0) = 77e^{-0.0339(90)} = 3.6 \approx 4$

$t = 100: f(0) = 77e^{-0.0339(100)} = 2.6 \approx 3$

| t	$f(t)$
0	77
10	55
20	39
30	28
40	20
50	14
60	10
70	7
80	5
90	4
100	3

MARKING SCHEME NOTES

Question 9 (d) [Scale 15C (0, 7, 10, 15)]

7: • Any one point identified
   • Graph of correct shape, even if no point correct or no point calculated
   • Accept candidates value of k
   NOTE: All graphs may not be the same, due to different values of A and k

10: • Three points correctly plotted, but graph incomplete or no graph
    NOTE: Do not accept straight line graph

Question 9 (e) (i)
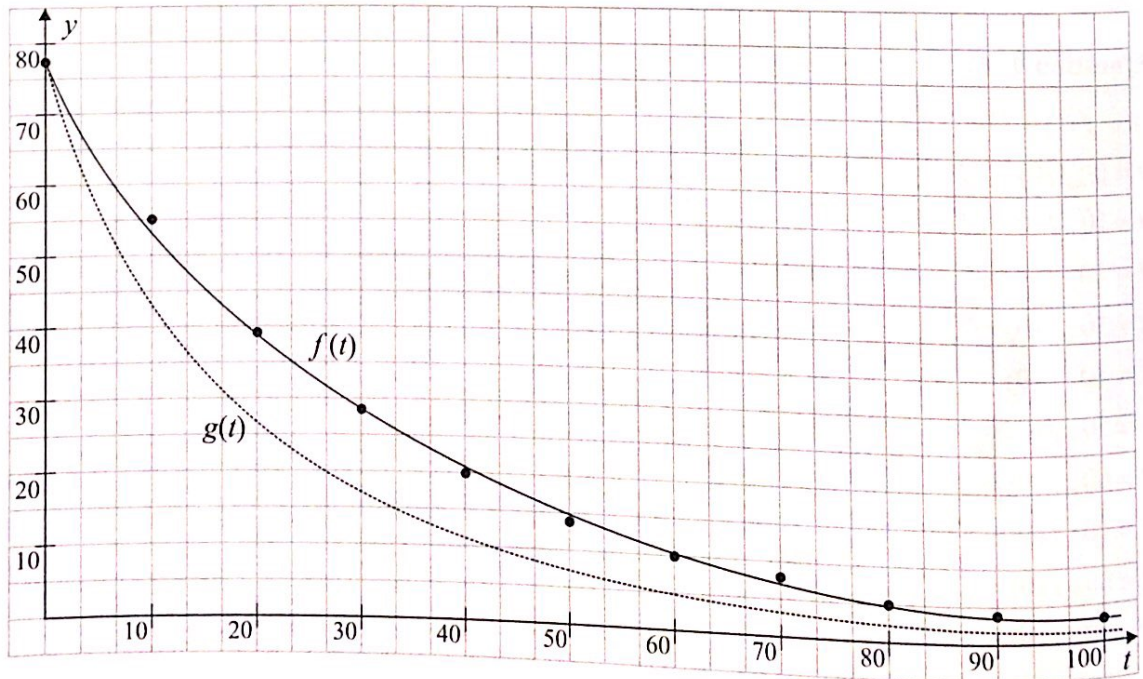
Question 9 (e) (ii)

$g(t) = 77e^{-0.05t}$

$m = -0.05$

This value of m gives a larger average rate of change for the cooling time required.

> **MARKING SCHEME NOTES**
> **Question 9 (e) (i) (ii) [Scale 5C (0, 3, 4, 5)]**
> 3: • Any attempt at similar graph
>   • No graph but correct deduction
> 4: • Correct graph plotted but graph incomplete, or no graph

Question 9 (f) (i)

$f(t) = 77e^{-0.0339t}$

$f'(t) = 77(-0.0339)e^{-0.0339t} = -2.6103e^{-0.0339t}$

$f'(1) = -2.6103e^{-0.0339(1)} = -2.52$ °C per minute

$f'(10) = -2.6103e^{-0.0339(10)} = -1.86$ °C per minute

Question 9 (f) (ii)

$f'(t) = -2.6103e^{-0.0339t}$

$f''(t) = -2.6103(-0.0339)e^{-0.0339t} = \dfrac{0.088}{e^{0.0339t}} > 0$ for all t

Therefore, the rate of change of $f(t)$ is always increasing over time.

> **MARKING SCHEME NOTES**
> **Question 9 (f) (i) [Scale 5C (0, 3, 4, 5)]**
> 3: • Indication of differentiation i.e. $\dfrac{dy}{dt}$, $\dfrac{dx}{dt}$ or $f'(t)$ (i.e. differentiation with respect to t)
>
>   • Treats e as x in differentiation
> 4: • One value of $\dfrac{dy}{dt}$ indicated
>
> **Question 9 (f) (ii) [Scale 5C (0, 3, 4, 5)]**
> 3: • Attempt at 2nd derivative
>   • Attempt at deduction from numerical values
> 4: • Shows 2nd derivative positive