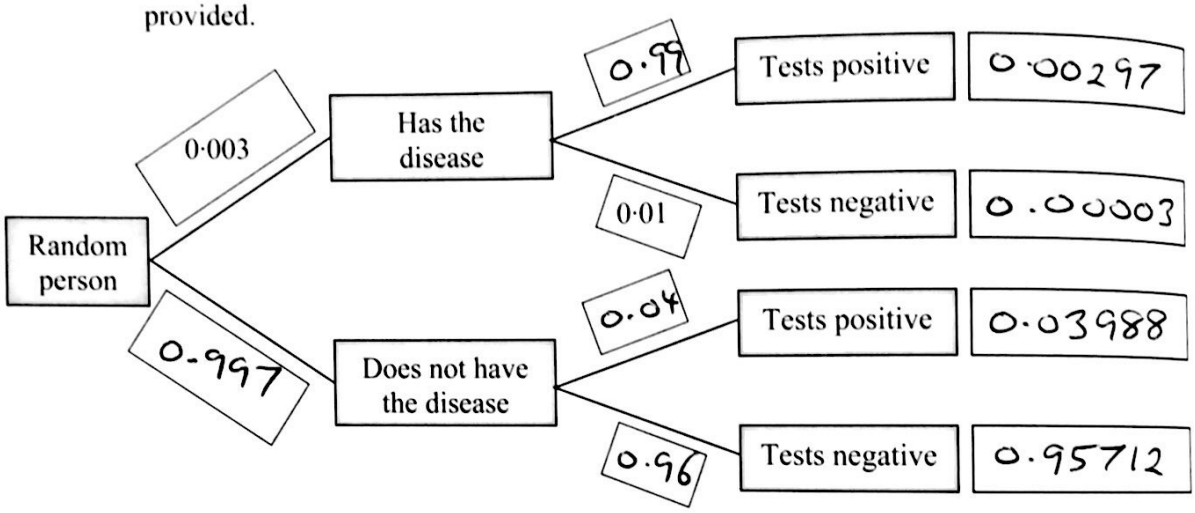
Question 8 (45 marks)

Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).

It is estimated that 0·3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

(a) **(i)** Write the probability associated with each branch of the tree diagram in the blank boxes provided.



(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

$$P(\text{positive}) = 0.00297 + 0.03988$$
$$= \boxed{0.04285}$$

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.

$$P(\text{has disease} \mid \text{positive}) = \frac{P(\text{has disease} \cap \text{positive})}{P(\text{positive})}$$
$$= \frac{0.00297}{0.04285}$$
$$= \boxed{0.0693}$$

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

Test is not very useful.
A person who tests positive only has the disease 7% of the time.

(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two new drugs, *A* and *B*, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug *A* was successful in the case of 296 patients. The company claims that Drug *A* is more successful in treating the condition than the generic drug.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

H_0 = Same success rate as generic drug

H_A = Not same success rate

$$\text{Margin of error} = 1.96\sqrt{\frac{\frac{296}{500}\left(1-\frac{296}{500}\right)}{500}} = 0.043$$

$\hat{p} = 0.592$

$0.549 \leq p \leq 0.635$

0.51 outside confidence interval

$\therefore$ Reject H_0. There is evidence to suggest success rate is better

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{500}} = 0.045$$

Greatest % success $= 0.51 + 0.045 = 0.555$

$0.555(500) = 277.5 \approx \boxed{277}$ people

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