Question 3 **(25 marks)**

A cubic function f is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1-k^2)x + k, \quad \text{where } k \text{ is a constant.}$$

(a) Show that $-k$ is a root of f.

$(-k)^3 + (1-k^2)(-k) + k$

$= -k^3 - k + k^3 + k$

$= 0 \quad \therefore$ root

(b) Find, in terms of k, the other two roots of f.

$(x+k)(x^2 + cx + 1) = x^3 + (1-k^2)x + k$

$\therefore x^3 + cx^2 + kx^2 + ckx + x + k = x^3 + (1-k^2)x + k$

<u>x^2 coeffs</u>

$c + k = 0 \quad \Rightarrow c = -k$

$\therefore (x+k)(x^2 - kx + 1) = 0$

$\therefore x^2 - kx + 1 = 0$

$\therefore x = \dfrac{k \pm \sqrt{k^2 - 4(1)(1)}}{2(1)} = \boxed{\dfrac{k \pm \sqrt{k^2-4}}{2}}$

(c) Find the set of values of k for which f has exactly one real root.

only 1 real root $\Rightarrow$ 2 complex roots

$\Rightarrow k^2 - 4 < 0$

$\Rightarrow k^2 < 4$

$\therefore \boxed{-2 < k < 2}$

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