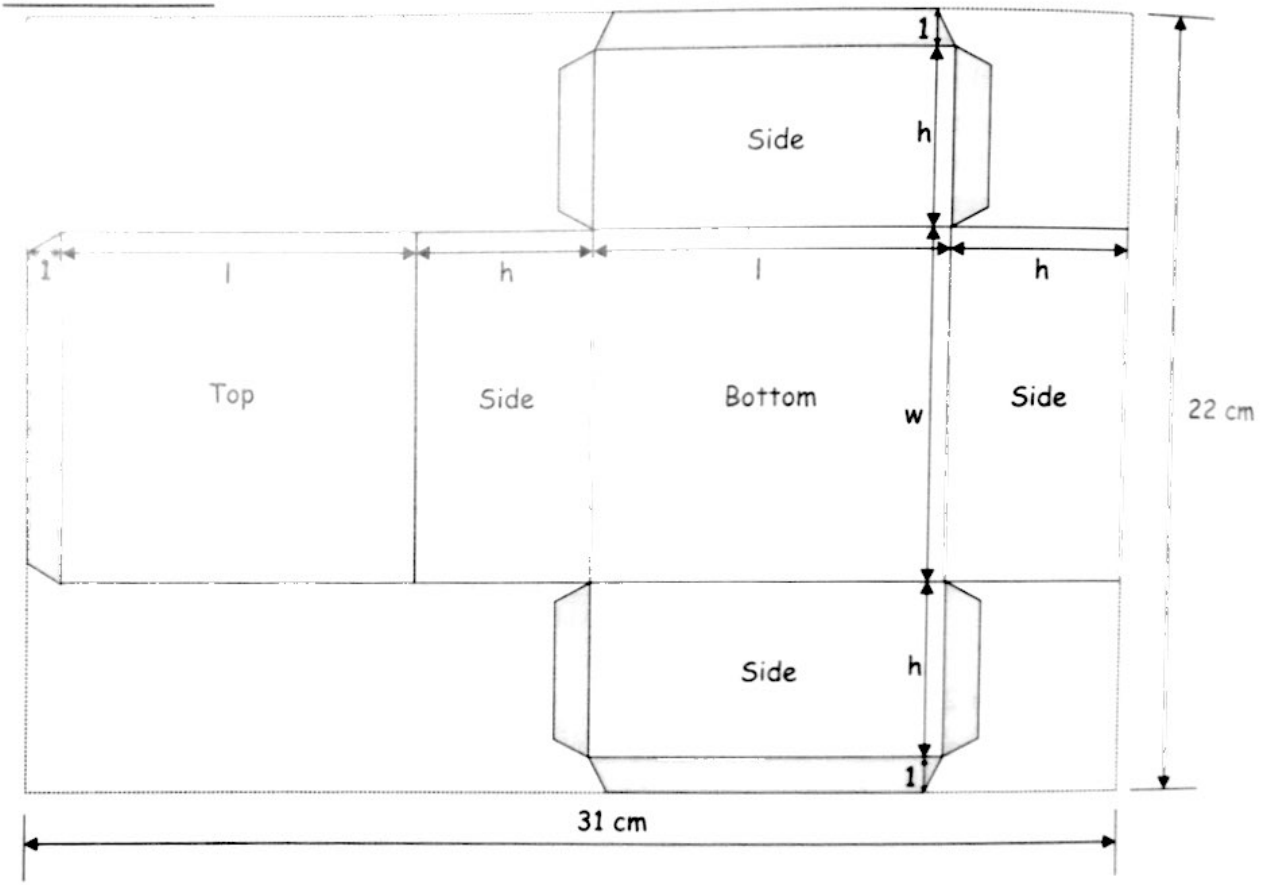
Question 7 (50 marks)

Question 7 (a)



$2h + 2l + 1 = 31$

$2l + 2h = 30$

$l + h = 15$

$l = (15 - h)$ cm

$2h + w + 2 = 22$

$2h + w = 20$

$w = (20 - 2h)$ cm

Question 7 (b)

$V = l \times b \times h = (15 - h)(20 - 2h)h$

Question 7 (c)

Square bottom: $l = w$

$15 - h = 20 - 2h$

$2h - h = 20 - 15$

$\therefore h = 5$ cm

$V = (15 - (5))(20 - 2(5))(5)$

$= (10)(10)(5)$

$= 500 \text{ cm}^3$

Question 7 (d)

$(15 - h)(20 - 2h)h = 500$ ← Form a cubic equation by putting the volume of the box equal to 500.

$(300 - 50h + 2h^2)h = 500$

$2h^3 - 50h^2 + 300h - 500 = 0$

$h^3 - 25h^2 + 150h - 250 = 0$

$h = 5$ is a solution of this cubic. Therefore, $(h - 5)$ is a linear factor. The other factor is a quadratic. Find the quadratic by lining up.

$h^3 - 25h^2 + 150h - 250 = (h-5)(h^2 + kh + 50)$

$h^3 - 25h^2 + 150h - 250 = h^3 + (k-5)h^2 + (50 - 5k)h - 250$

$\therefore -25 = k - 5 \Rightarrow k = -20$

$h^3 - 25h^2 + 150h - 250 = (h-5)(h^2 - 20h + 50) = 0$

$h^2 - 20h + 50 = 0$ ← Solve the quadratic using the formula.

$a = 1,\ b = -20,\ c = 50$

$$h = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(50)}}{2(1)}$$

$$= \frac{20 \pm \sqrt{400 - 200}}{2}$$

$$= \frac{20 \pm \sqrt{200}}{2}$$

$$= \frac{20 \pm 10\sqrt{2}}{2}$$

$$= 10 \pm 5\sqrt{2}$$

$= \cancel{17.1},\ 2.9\text{ cm}$ [Discard the solution of $h = 17{\cdot}1$ cm. The length l of the box is equal to $(15 - h)$. The length is not long enough to accommodate a value of 17·1 cm.]

Formulae and Tables Book
Algebra: Roots of the quadratic equation
$ax^2 + bx + c = 0$ [page 20]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Answer: $h = 2{\cdot}9$ cm

Question 7 (e)