Question 5 (c)

$$\frac{dy}{dx}=0 \Rightarrow = 1-\frac{1}{2\sqrt{x+6}}=0$$

> Find turning points (local maxiumum/minimum)
>
> Put $\frac{dy}{dx}=0$ and solve for x

$$1=\frac{1}{2\sqrt{x+6}}$$

$2\sqrt{x+6}=1 \leftarrow$ Square both sides.

$$4(x+6)=1$$

$$4x+24=1$$

$$4x=-23$$

$$x=-\tfrac{23}{4}$$

$$x=-\tfrac{23}{4}: y=x-\sqrt{x+6}=-\frac{23}{4}-\sqrt{-\frac{23}{4}+6}=-\frac{23}{4}-\sqrt{\frac{1}{4}}=-\frac{23}{4}-\frac{1}{2}=-\frac{25}{4}$$

Turning point $(-\frac{23}{4}, -\frac{25}{4})$

> **Marking Scheme Notes**
>
> **Question 5 (c) [Scale 10C (0, 4, 8, 10)]**
>
> **4:** • Differentiation equals 0
>
> **8:** • Finds x value
>
> **Note** 1: A linear equation from $f'(x)$ gets low partial at most
>
> **Note** 2: Must put $f'(x) = 0$ in (c) to get any marks
>
> **Note** 3: $f'(x)$ only and $f''(x)$ only: no credit

Question 6 (25 marks)

Question 6 (a)

(i) **Bank A**: Monthly interest rate $r_M = 0{\cdot}35\% \Rightarrow i_M = 0{\cdot}0035$

$(1 + i_M)^{12} = (1 + i_A)$, where i_A is the annual interest rate

$(1 + 0{\cdot}0035)^{12} = (1 + i_A)$

$(1{\cdot}0035)^{12} = (1 + i_A)$

$1{\cdot}042818 = 1 + i_A$

$i_A = 0{\cdot}042818$

$\therefore$ Annual percentage rate (APR) $r_A = 4{\cdot}28\%$ [Given to 3 significant figures]

(ii) **Bank B**: Annual interest rate $r_A = 4{\cdot}5\% \Rightarrow i_A = 0{\cdot}045$

Monthly interest rate: $i_M = ?$, $r_M = ?$

$(1 + i_M)^{12} = (1 + 0{\cdot}045)$

$(1 + i_M)^{12} = (1{\cdot}045)$

$i_M = 1{\cdot}045^{\frac{1}{12}} - 1 = 0{\cdot}0036748$

$\therefore r_M = 0{\cdot}367\%$ [Given to 3 significant figures]

Marking Scheme Notes

Question 6 (a) (i) (ii) [Scale 10C (0, 4, 8, 10)]

4: • Correct formula in either part
• Correct substitution in incorrect formula

8: • Any one section correct

Note: Rate as 0·367% or 0·00367 gets High Partial.

Question 6 (b)

Formulae and Tables Book
Financial mathematics: Amortisation - mortgages and loans
(equal repayments at equal intervals) [page 31]

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

t = Time period (in years)
i = (Annual) rate of interest expressed as a decimal
A = (Annual) repayment amount
P = Principal
Note: The time period can be months or weeks instead of years provided the interest rate is given for that time period.

Method 1: Use the amortisation formula

Time period: Months
$r_M = 0{\cdot}35\% \Rightarrow i_M = 0{\cdot}0035$
$t = 10 \times 12 = 120$ months
$P = €80\,000$

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$
$$= 80\,000\frac{0\cdot0035(1\cdot0035)^{120}}{(1\cdot0035)^{120} - 1}$$
$$= €817\cdot59 \approx €818$$

Method 2: Use a geometric series

$$\frac{A}{1\cdot0035} + \frac{A}{1\cdot0035^2} + \ldots + \frac{A}{1\cdot0035^{120}} = 80\,000$$
$$A\left[\frac{1}{1\cdot0035} + \frac{1}{1\cdot0035^2} + \ldots + \frac{1}{1\cdot0035^{120}}\right] = 80\,000$$
$$A\left[\frac{\frac{1}{1\cdot0035}\left(1 - \left(\frac{1}{1\cdot0035}\right)^{120}\right)}{1 - \frac{1}{1\cdot0035}}\right] = 80\,000$$
$$\therefore A = \frac{80\,000\left(1 - \frac{1}{1\cdot0035}\right)}{\frac{1}{1\cdot0035}\left(1 - \left(\frac{1}{1\cdot0035}\right)^{120}\right)} = €81\square\cdot5\square \approx €818$$

Formulae and Tables Book
Sequences and series:
Geometric series [page 22]

$$S_n = \frac{a(1-r^n)}{1-r}$$