

QUESTION 8 (50 MARKS)

Question 8 (a) (i)

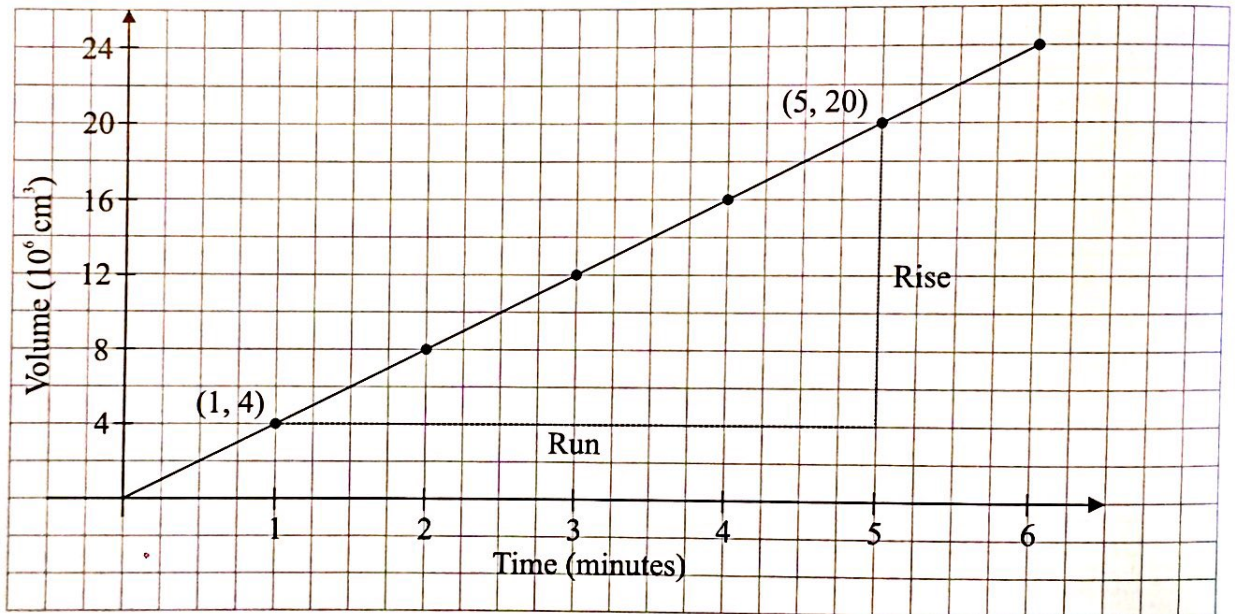
Volume $V = 4 \times 10^6 \text{ cm}^3$ per minute

Time $t = 1$ minute $\Rightarrow V = 4 \times 10^6 \text{ cm}^3$

Time $t = 2$ minutes $\Rightarrow V = 4 \times 10^6 \times 2 = 8 \times 10^6 \text{ cm}^3$

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)	4	8	12	16	20	24

Question 8 (a) (ii)



Question 8 (a) (iii)

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{20 \times 10^6 - 4 \times 10^6}{5 - 1} = \frac{16 \times 10^6}{4} = 4 \times 10^6$$

$$V = 4 \times 10^6 t \quad [y = mx]$$

MARKING SCHEME NOTES

Question 8 (a) (i) [Scale 5B (0, 2, 5)]

2: • One correct box

Question 8 (a) (ii) [Scale 5B (0, 2, 5)]

0: • Bar chart

2: • At least two points plotted

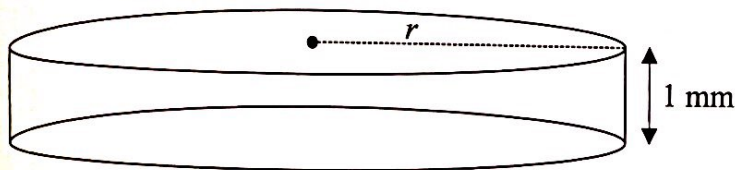
Question 8 (a) (iii) [Scale 5B (0, 2, 5)]

2: • Incomplete equation for volume

• $V =$ any function of t

• Attempt at finding slope

Question 8 (b) (i)



$$V = \pi r^2 h = \pi r^2 (0.1) = \frac{1}{10} \pi r^2 \text{ cm}^3$$

Question 8 (b) (ii)

$$V = 4 \times 10^6 t \Rightarrow \frac{dV}{dt} = 4 \times 10^6$$

$$V = \frac{1}{10} \pi r^2 \Rightarrow \frac{dV}{dt} = \frac{1}{5} \pi r \times \frac{dr}{dt} = 4 \times 10^6$$

$$r = 50 \text{ m} = 5000 \text{ cm} : 4 \times 10^6 = \frac{1}{5} \pi (5000) \times \frac{dr}{dt}$$

$$4000 = \pi \times \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{4000}{\pi} \text{ cm/minute} = 1273.3 \text{ cm/minute}$$

FORMULAE AND TABLES BOOK
Surface area and volume:
Cylinder [page 10]

A diagram of a cylinder. A horizontal arrow from the center of the top circular face to the edge is labeled 'r'. A vertical double-headed arrow on the left side of the cylinder is labeled 'h'.

$$A = 2\pi r h$$

$$V = \pi r^2 h$$

MARKING SCHEME NOTES

Question 8 (b) (i) [Scale 5B (0, 2, 5)]

- 2: • Correct volume formula
 • Converting mm to cm

Question 8 (b) (ii) [Scale 10D (0, 2, 5, 8, 10)]

- 2: • Mentions a relevant rate of change
 5: • Gets $\frac{dr}{dt}$ from $\frac{dV}{dr}$ and $\frac{dV}{dt}$
 • Writing down chain rule
 8: • Substitution of values

Question 8 (c)

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$$

$$\frac{1}{5} \pi r \times \frac{dr}{dt} = 4 \times 10^6 \Rightarrow \frac{dr}{dt} = \frac{5 \times 4 \times 10^6}{\pi r}$$

$$\therefore \frac{dA}{dt} = 2\pi r \times \frac{5 \times 4 \times 10^6}{\pi r} = 4 \times 10^7 \text{ cm}^2/\text{minute}$$

or

$$\frac{1}{10} \pi r^2 = (4 \times 10^6) t$$

$$A = \pi r^2 = (4 \times 10^7) t$$

$$\therefore \frac{dA}{dt} = 4 \times 10^7 \text{ cm}^2/\text{minute}$$

FORMULAE AND TABLES BOOK
Length and area:
Circle [page 8]

A diagram of a circle. A horizontal arrow from the center to the edge is labeled 'r'. A curved arrow along the circumference is labeled 'l'.

$$l = 2\pi r$$

$$A = \pi r^2$$

MARKING SCHEME NOTES

Question 8 (c) [Scale 10C (0, 4, 8, 10)] Note: two solutions

1st solution

4: • Mentions relevant rate of change

8: • States chain rule i.e. $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

or

2nd solution

4: • Effort to establish value of A

8: • A in terms of t

NOTE: Must use calculus to get any credit

Question 8 (d)

$$r = 1 \text{ km} = 10^5 \text{ cm}$$

$$A = \pi r^2 = \pi(10^5)^2 = 10^{10} \pi \text{ cm}^2$$

$$\frac{dA}{dt} = 4 \times 10^7 \text{ cm}^2/\text{minute}$$

$$\text{Time } t = \frac{10^{10} \pi \text{ cm}^2}{4 \times 10^7 \text{ cm}^2/\text{minute}} = 785.4 \text{ minutes} \approx 13 \text{ hours}$$

or

$$r = 1 \text{ km} = 10^5 \text{ m}$$

$$\frac{dr}{dt} = \frac{20 \times 10^6}{\pi r}$$

$$\pi r dr = 20 \times 10^6 dt$$

$$\int_0^{10^5} \pi r dr = \int_0^t 20 \times 10^6 dt$$

$$\left[\frac{\pi r^2}{2} \right]_0^{10^5} = [20 \times 10^6 t]_0^t$$

$$\frac{\pi(10^5)^2}{2} - \frac{\pi(0)^2}{2} = 20 \times 10^6 t$$

$$10^{10} \pi = 40 \times 10^6 t$$

$$10^4 \pi = 40t$$

$$\therefore t = \frac{10^4 \pi}{40} = 250\pi \text{ minutes} = \frac{250\pi}{60} \text{ hours} \approx 13 \text{ hours}$$

FORMULAE AND TABLES BOOK
Calculus: Integrals [page 26]

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

MARKING SCHEME NOTES

Question 8 (d) [Scale 10C (0, 4, 8, 10)]

4: • r in centimetres

• Effort at expression of area

8: • Correct expression for time