Question 3 (25 marks)

Question 3 (a)

$A(4, -1)$, $B(7, t)$

$$m = \frac{t-(-1)}{7-4} = \frac{t+1}{3}$$

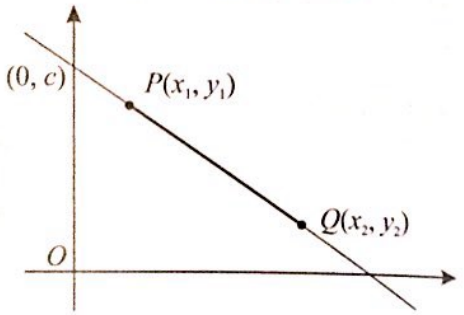
$$l_1 : 3x - 4y - 12 = 0 \Rightarrow m = \frac{3}{4}$$

$$\therefore m_\perp = -\frac{4}{3}$$

$$\therefore \frac{t+1}{3} = -\frac{4}{3}$$

$$t + 1 = -4 \Rightarrow t = -5$$

Formulae and Tables Book
Co-ordinate geometry: Line
Slope of PQ [page 18]



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Marking Scheme Notes
Question 3 (a) [Scale 10D (0, 2, 5, 8, 10)]
2: • Slope AB or l_1
5: • Both slopes found
8: • Slopes linked to perpendicularity

Question 3 (b)

Call this distance d_1.

$l_1 : 3x - 4y - 12 = 0$

$P(10, k)$

$$d_1 = \frac{|3(10) - 4k - 12|}{\sqrt{3^2 + (-4)^2}} = \frac{|18 - 4k|}{5}$$

Finding the slope from the equation of a line:

$l : ax + by + c = 0$

$$m = -\frac{a}{b} \Rightarrow m_\perp = \frac{b}{a}$$

Marking Scheme Notes
Question 3 (b) [Scale 10C (0, 4, 8, 10)]
4: • Relevant formula with some correct substitution
8: • Substitution into formula fully correct

Question 3 (c) (i)

If $P(10, k)$ lies on the bisector of the angles between l_1 and l_2, it is the same perpendicular distance from each line.

$l_2 : 5x + 12y - 20 = 0$

$P(10, k)$

$$d_2 = \frac{|5(10) + 12k - 20|}{\sqrt{5^2 + 12^2}} = \frac{|30 + 12k|}{13}$$

$$d_1 = d_2 \Rightarrow \frac{|18 - 4k|}{5} = \frac{|30 + 12k|}{13}$$

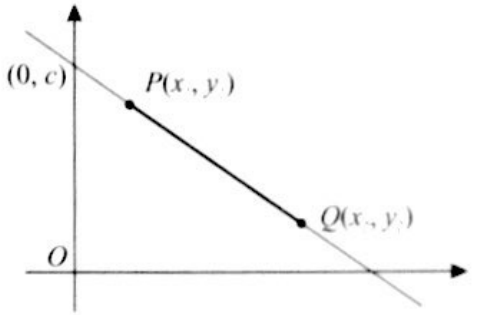
$$13(18 - 4k) = \pm 5(30 + 12k)$$

$$13(9 - 2k) = \pm 5(15 + 6k)$$

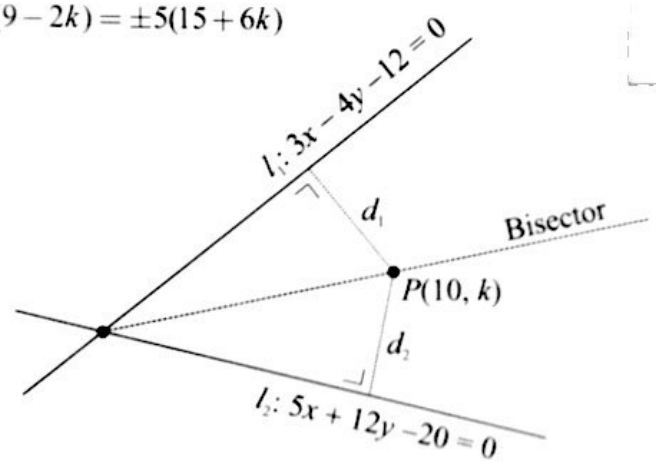
Formulae and Tables Book

Co-ordinate geometry: Line

Distance from (x_1, y_1) to the line $ax + by + c = 0$ [page 19]



$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$



$13(9 - 2k) = 5(15 + 6k)$

$117 - 26k = 75 + 30k$

$42 = 56k$

$\therefore k = \frac{42}{56} = \frac{3}{4}$

$13(9 - 2k) = -5(15 + 6k)$

$117 - 26k = -75 - 30k$

$192 = -4k$

$\therefore k = -48$

Question 3 (c) (ii)

$$d_1 = \frac{\left|18 - 4\left(\frac{3}{4}\right)\right|}{5} = \frac{|18 - 3|}{5} = \frac{15}{5} = 3$$

Marking Scheme Notes

Question 3 (c) (i) (ii) [Scale 5D (0, 2, 3, 4, 5)]

2: • Relevant formula with some correct substitution

3: • One value for k found
• Work indicating two values for k

4: • Both values of k
• Positive value for k evaluated and distance calculated