Question 5 (25 marks)

Question 5 (a)

You are asked to prove an identity. You have to show that the left-hand side (LHS) is equal to the right-hand side (RHS).

Turn all trigonometric functions into sin and cos.

Formulae and Tables Book
Trigonometry (page 13/14)

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

LHS

$$\tan(A+B)$$
$$= \frac{\sin(A+B)}{\cos(A+B)}$$
$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

RHS

$$\frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$= \frac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{1 - \dfrac{\sin A}{\cos A}\times\dfrac{\sin B}{\cos B}}$$
$$= \frac{\left(\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}\right)\times\cos A\cos B}{\left(1 - \dfrac{\sin A}{\cos A}\times\dfrac{\sin B}{\cos B}\right)\times\cos A\cos B}$$
$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

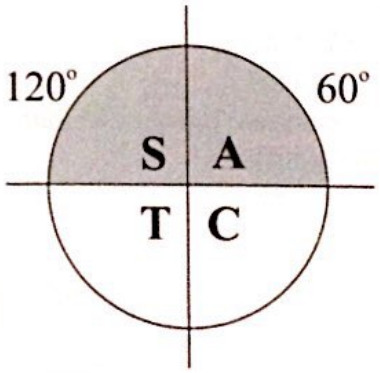
$$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Marking Scheme Notes
Question 5 (a) [Scale 15D (0, 4, 7, 11, 15)]
4: • Tan function in terms of Sine and Cosine
7: • $\sin(A+B)$ or $\cos(A+B)$ expanded
• Numerator or denominator in fraction form
11: • Numerator and denominator divided by $\cos A\cos B$
• Both numerator and denominator expressed in form of a single fraction

Question 5 (b)

$$\sin(3x) = \frac{\sqrt{3}}{2}$$

Reference angle $\alpha = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = 60°$



$$3x = \begin{cases} 60°, 420°, 780° \text{ [First quadrant]} \\ 120°, 480°, 840° \text{ [Second quadrant]} \end{cases}$$

$$x = \begin{cases} 20°, 140°, 260° \\ 40°, 160°, 280° \end{cases}$$

$$\therefore x = 20°, 40°, 140°, 160°, 260°, 280°$$

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