

Answer all six questions from this section.

Question 1 *Euler*

(25 marks)

- (a)  $(-4 + 3i)$  is one root of the equation  $az^2 + bz + c = 0$ , where  $a, b, c \in \mathbb{R}$ , and  $i^2 = -1$ . Write the other root.

$$-4 - 3i$$

- (b) Use De Moivre's Theorem to express  $(1+i)^8$  in its simplest form.

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$$

$$\therefore \left[ \sqrt{2} (\cos 45^\circ + i \sin 45^\circ) \right]^8$$

$$= \sqrt{2}^8 (\cos 360^\circ + i \sin 360^\circ)$$

$$= 16 (1 + 0i)$$

$$= \boxed{16}$$

- (c)  $(1+i)$  is a root of the equation  $z^2 + (-2+i)z + 3 - i = 0$ .

Find its other root in the form  $m+ni$ , where  $m, n \in \mathbb{R}$ , and  $i^2 = -1$ .

$$z^2 - Sz + P = 0$$

$$S = -(-2+i) = 1+i + z_2$$

$$\therefore z_2 = 2-i - 1-i$$

$$= \boxed{1-2i}$$