May 17

Question 4 **(25 marks)**

(a) Prove by induction that $8^n - 1$ is divisible by 7 for all $n \in \mathbb{N}$.

Let $n = 1$

$8^1 - 1 = 8 - 1 = 7$ $\therefore$ divisible by 7

$\therefore$ true for $n = 1$

Assume true for $n = k$

$\Rightarrow \boxed{8^k - 1 = 7m}$ $m \in N$

Let $n = k + 1$

$\Rightarrow 8^{k+1} - 1$

$= 8^k(8^1) - 1$

$= (7m + 1)(8) - 1$

$= 7(8m) + 8 - 1$

$= 7(8m + 1)$ divisible by 7

$\therefore$ True for $n = k$ $\Rightarrow$ true for $n = k + 1$

$\therefore$ True for all $n \in N$ by induction.

(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q

(i) $\log_a \frac{8}{3}$

$= \log_a 8 - \log_a 3$

$= \log_a 2^3 - \log_a 3$

$= 3\log_a 2 - \log_a 3 = \boxed{3p - q}$

(ii) $\log_a \frac{9a^2}{16}$.

$= \log_a 9a^2 - \log_a 16$

$= \log_a (3a)^2 - \log_a 2^4$

$= 2\log_a 3a - 4\log_a 2$

$= 2[\log_a 3 + \log_a a] - 4p$

$= 2(q + 1) - 4p$

$= \boxed{2q + 2 - 4p}$

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