

Oct 16

Question 6

(25 marks)

(a) Differentiate the function $(2x + 4)^2$ from first principles, with respect to x .

$$f(x) = y = 4x^2 + 16x + 16$$

$$f(x+h) = 4(x+h)^2 + 16(x+h) + 16$$

$$= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16$$

$$-f(x) = 4x^2 + 16x + 16$$

$$f(x+h) - f(x) = 8hx + 4h^2 + 16h$$

$$\frac{f(x+h) - f(x)}{h} = 8x + 4h + 16$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \boxed{8x + 16}$$

(b) (i) If $y = x \sin\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$.

$$y = x \sin(x^{-1})$$

$$\frac{dy}{dx} = x \cos(x^{-1}) (-x^{-2}) + \sin(x^{-1}) (1)$$

$$= -\frac{1}{x} \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$$

- (ii) Find the slope of the tangent to the curve $y = x \sin\left(\frac{1}{x}\right)$, when $x = \frac{4}{\pi}$.
Give your answer correct to two decimal places.

$$\text{slope} = \frac{dy}{dx}$$

$$= -\frac{\pi}{4} \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)$$

$$= \boxed{0.15}$$