

Answer all three questions from this section.

Question 7 Mark 7

(40 marks)

- (a) (i) Air is pumped into a spherical exercise ball at the rate of 250 cm^3 per second. Find the rate at which the radius is increasing when the radius of the ball is 20 cm. Give your answer in terms of π .

$$\begin{aligned} \frac{dV}{dt} &= 250 & V &= \frac{4}{3}\pi r^3 \\ \frac{dr}{dt} &= \frac{dr}{dV} \times \frac{dV}{dt} & \frac{dV}{dr} &= 4\pi r^2 \\ &= \frac{1}{4\pi r^2} (250) \\ r = 20 &\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi (20)^2} (250) = \boxed{\frac{5}{32\pi} \text{ cm/s}} \\ & & &= \boxed{0.49 \text{ cm/s}} \end{aligned}$$

- (ii) Find the rate at which the surface area of the ball is increasing when the radius of the ball is 20 cm.

$$\begin{aligned} SA &= 4\pi r^2 = A & \frac{dA}{dr} &= 8\pi r \\ \frac{dA}{dt} &= \frac{dA}{dr} \times \frac{dr}{dt} \\ &= 8\pi \times \left(\frac{1}{4\pi r^2}\right) (250) \\ r = 20 &\Rightarrow \frac{dA}{dt} = 8 \left(\frac{1}{4(20)}\right) (250) \\ & & &= \boxed{25 \text{ cm}^2/\text{s}} \end{aligned}$$

- (b) The inflated ball is kicked into the air from a point O on the ground. Taking O as the origin, $(x, f(x))$ approximately describes the path followed by the ball in the air, where

$$f(x) = -x^2 + 10x$$

and both x and $f(x)$ are measured in metres.

- (i) Find the values of x when the ball is on the ground.

$$f(x) = 0$$

$$\Rightarrow -x^2 + 10x = 0$$

$$\Rightarrow x^2 - 10x = 0$$

$$\Rightarrow (x)(x - 10) = 0$$

$$\boxed{x = 0 \text{ m}} \quad \text{or} \quad \boxed{x = 10 \text{ m}}$$

- (ii) Find the average height of the ball above the ground, during the interval from when it is kicked until it hits the ground again.

$$\text{Ave height} = \frac{1}{10-0} \int_0^{10} (-x^2 + 10x) dx$$

$$= \frac{1}{10} \left[-\frac{x^3}{3} + \frac{10x^2}{2} \right]_0^{10}$$

$$= \frac{1}{10} \left(-\frac{10^3}{3} + \frac{10(10)^2}{2} - (0) \right)$$

$$= \boxed{\frac{50}{3} \text{ m}} = \boxed{16.67 \text{ m}}$$