Question 8 **(45 marks)**

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(a) Find the period and range of $h(t)$.

Period: $= \frac{2\pi}{\frac{\pi}{6}} =$ 12 hours

Range: $[1{\cdot}6 - 1{\cdot}5,\ 1{\cdot}6 + 1{\cdot}5] = [0{\cdot}1\text{m},\ 3{\cdot}1\text{m}]$

(b) Find the maximum height of the water in the port.

Max height = 3·1 m

(c) Find the rate at which the height of the water is changing when $t = 2$, correct to two decimal places. Explain your answer in the context of the question.

Rate: $\frac{dh}{dt} = -1{\cdot}5\sin\left(\frac{\pi}{6}t\right)\left(\frac{\pi}{6}\right)$

$t = 2 \Rightarrow -1{\cdot}5\sin\left(\frac{\pi}{6}(2)\right)\left(\frac{\pi}{6}\right)$

$= -0{\cdot}68\text{ m/h}$

Explanation: The tide is going out. The water is going down by 0·68 m per hour at 2am.

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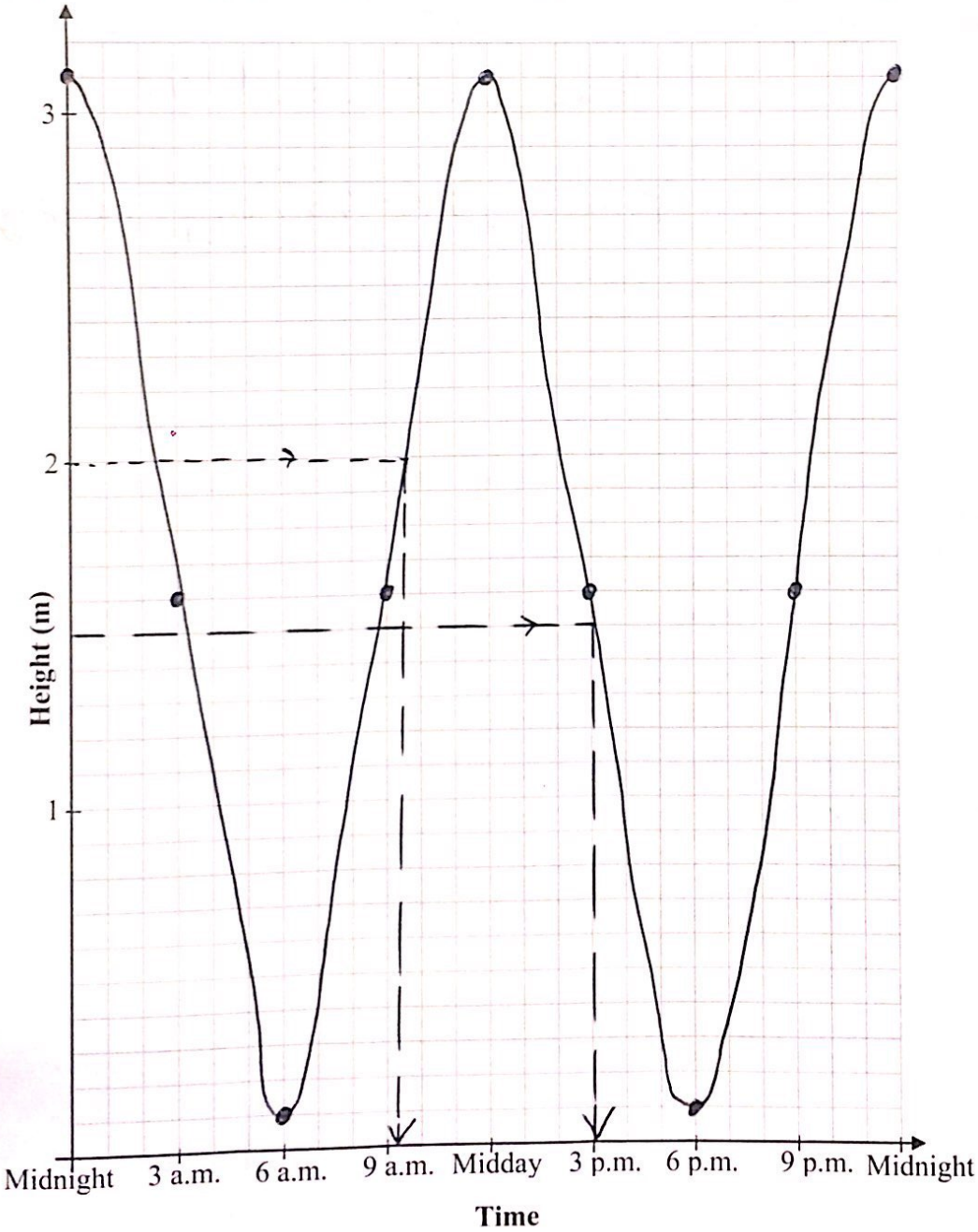
(d) (i) On a particular day the high tide occurred at midnight (i.e. $t = 0$). Use the function to complete the table and show the height, $h(t)$, of the water between midnight and the following midnight.

$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
t (hours)	0	3	6	9	12	15	18	21	24
$h(t)$ (m)	3·1	1·6	0·1	1·6	3·1	1·6	0·1	1·6	3·1

(ii) Sketch the graph of $h(t)$ between midnight and the following midnight.



(e) Find, from your sketch, the difference in water height between low tide and high tide.

Difference = 3.1 − 0.1

= 3 m

(f) A fully loaded barge enters the port, unloads its cargo and departs some time later.
The fully loaded barge requires a minimum water level of 2 m.
When the barge is unloaded it only requires 1·5 m.
Use your graph to estimate the **maximum** amount of time that the barge can spend in port, without resting on the sea-bed.

From graph: enter port around 9.30

leave port around 15.15

∴ Time = 5 hr 45 min roughly

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