

Question 9

(50 marks)

Data on earnings were published for a particular country. The data showed that the annual income of people in full-time employment was normally distributed with a mean of €39 400 and a standard deviation of €12 920.

SEC Set A
2016 P2

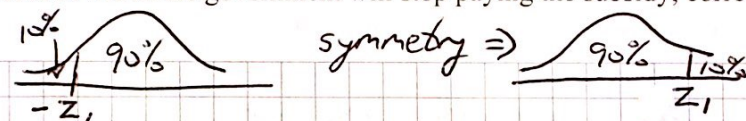
- (a) (i) The government intends to impose a new tax on incomes over €60 000. Find the percentage of full-time workers who will be liable for this tax, correct to one decimal place.

$$Z = \frac{x - \mu}{\sigma} = \frac{60000 - 39400}{12920} = 1.59$$

$$P(Z > 1.59) = 1 - P(Z < 1.59)$$

$$= 1 - 0.9441 = 0.0559 \approx \boxed{5.6\%}$$

- (ii) The government will also provide a subsidy to the lowest 10 % of income earners. Find the level of income at which the government will stop paying the subsidy, correct to the nearest euro.



symmetry \Rightarrow

$$P(Z < z_1) = 0.1$$

$$\therefore z_1 = 1.28 \quad \Rightarrow \quad -z_1 = -1.28$$

$$Z = \frac{x - \mu}{\sigma} \Rightarrow -1.28 = \frac{x - 39400}{12920} \Rightarrow \boxed{x = €22\,862}$$

- (iii) Some time later a research institute surveyed a sample of 1000 full-time workers, randomly selected, and found that the mean annual income of the sample was €38 280. Test the hypothesis, at the 5 % level of significance, that the mean annual income of full-time workers has changed since the national data were published. State the null hypothesis and the alternative hypothesis. Give your conclusion in the context of the question.

$$H_0: \mu = 39400 \quad (\text{has not changed})$$

$$H_A: \mu \neq 39400 \quad (\text{has changed})$$

$$\text{Margin of error} = 1.96 \sigma_{\bar{x}} = 1.96 \frac{\sigma}{\sqrt{n}} = 1.96 \left(\frac{12920}{\sqrt{1000}} \right)$$

$$= 800.79$$

Confidence interval:

$$37479.21 \leq \mu \leq 39080.79$$

39000 is outside this interval

\therefore reject H_0 ... reason to think mean income has changed.

- (b) The research institute surveyed 400 full-time farmers, randomly selected from all the full-time farmers in the country, and found that the mean income for the sample was €26 974 and the standard deviation was €5120. Assuming that annual farm income is normally distributed in this country, create a 95 % confidence interval for the mean income of full-time farmers.

$$\text{Margin of error} = 1.96 \frac{\sigma}{\sqrt{n}} = 1.96 \left(\frac{5120}{\sqrt{400}} \right) = 501.76$$

$$\therefore 26974 - 501.76 \leq \mu \leq 26974 + 501.76$$

$$\therefore \boxed{26472.24 \leq \mu \leq 27475.76}$$

- (c) It is known that data on farm size are not normally distributed. The research institute could take many large random samples of farm size and create a sampling distribution of the means of all these samples. Give one reason why they might do this.

The sample means would be normally distributed.

- (d) The research institute also carried out a survey into the use of agricultural land. n farmers were surveyed. If the margin of error of the survey was 4.5 %, find the value of n .

$$\frac{1}{\sqrt{n}} = 0.045$$

$$\therefore \frac{1}{0.045} = \sqrt{n}$$

$$\therefore n = 493.827$$

$$\text{ie } \boxed{n = 494}$$