

Question 2

(25 marks)

$z = -\sqrt{3} + i$ , where  $i^2 = -1$ .

- (a) Use De Moivre's Theorem to write  $z^4$  in the form  $a + b\sqrt{c}i$ , where  $a, b$ , and  $c \in \mathbb{Z}$ .

ref angle =  $\tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$   
 $\therefore \theta = 150^\circ$

$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = 2$

$\therefore z = 2 (\cos 150^\circ + i \sin 150^\circ)$

$\therefore z^4 = 2^4 (\cos 600^\circ + i \sin 600^\circ)$   
 $= \boxed{-8 - 8\sqrt{3}i}$

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- (b) The complex number  $w$  is such that  $|w| = 3$  and  $w$  makes an angle of  $30^\circ$  with the positive sense of the real axis. If  $t = zw$ , write  $t$  in its simplest form.

$w = 3 (\cos 30^\circ + i \sin 30^\circ)$

$\therefore t = 2 (\cos 150^\circ + i \sin 150^\circ) (3) (\cos 30^\circ + i \sin 30^\circ)$   
 $= 6 (\cos 180^\circ + i \sin 180^\circ)$   
 $= 6 (-1 + 0i)$   
 $= \boxed{-6}$