**Question 3** **(25 marks)**

**(a)** Differentiate $\frac{1}{3}x^2 - x + 3$ from first principles with respect to $x$.

$$f(x) = \frac{1}{3}x^2 - x + 3$$

$$f(x+h) = \frac{1}{3}(x+h)^2 - (x+h) + 3$$

$$= \frac{1}{3}(x^2 + 2hx + h^2) - x - h + 3$$

$$= \frac{1}{3}x^2 + \frac{2h}{3}x + \frac{h^2}{3} - x - h + 3$$

$$-f(x) = -\frac{1}{3}x^2 \qquad +x \qquad -3$$

$$f(x+h) - f(x) = \frac{2h}{3}x + \frac{h^2}{3} - h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2}{3}x + \frac{h}{3} - 1$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \boxed{\frac{2}{3}x - 1}$$

**(b)** $f(x) = \ln(3x^2 + 2)$ and $g(x) = x + 5$, where $x \in \mathbb{R}$.

Find the value of the derivative of $f(g(x))$ at $x = \frac{1}{4}$.

Give your answer correct to 3 decimal places.

$$f(g(x)) = \ln\left(3(x+5)^2 + 2\right)$$

$$= \ln\left(3(x^2 + 10x + 25) + 2\right)$$

$$= \ln\left(3x^2 + 30x + 77\right)$$

$$f'(g(x)) = \frac{1}{3x^2 + 30x + 77}(6x + 30)$$

$$x = \frac{1}{4} \Rightarrow \frac{1}{3\left(\frac{1}{4}\right)^2 + 30\left(\frac{1}{4}\right) + 77}\left(6\left(\frac{1}{4}\right) + 30\right)$$

$$= \boxed{0.372}$$

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