Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7

(55 marks)

Sometimes it is possible to predict the future population in a city using a function.
The population in Sapphire City, over time, can be predicted using the following function:

$$p(t) = Se^{0.1t} \times 10^6.$$

The population in Avalon, over time, can be predicted using the following function:

$$q(t) = 3.9e^{kt} \times 10^6.$$

In the functions above, t is time, in years; $t = 0$ is the beginning of 2010; and both S and k are constants.

(a) The population in Sapphire City at the beginning of 2010 is 1 100 000 people.
Find the value of S.

$1100000 = Se^{0} \times 10^6$

$\frac{1100000}{10^6} = S = 1.1$

(b) Find the predicted population in Sapphire City at the beginning of 2015.

$P = 1.1e^{0.1(5)} \times 10^6$

$= 1,813,593$

(c) Find the predicted change in the population in Sapphire City during 2015.

$p(6) = 1.1e^{0.1(6)} \times 10^6$

$= 2004331$

$\therefore$ change $= 190738$

(d) The predicted population in Avalon at the beginning of 2011 is 3 709 795 people.
Write down and solve an equation in k to show that $k = -0{\cdot}05$, correct to 2 decimal places.

$$3\,709\,795 = 3{\cdot}9e^{k(1)} \times 10^6$$

$$\frac{3{\cdot}709795}{3{\cdot}9} = e^k$$

$$\ln 0{\cdot}95122295 = \boxed{k = -0{\cdot}05}$$

(e) Find the year during which the populations in both cities will be equal.

$$3{\cdot}9e^{0{\cdot}1t} \times 10^6 = 1{\cdot}1e^{0{\cdot}1t} \times 10^6$$

$$\frac{3{\cdot}9}{1{\cdot}1} = \frac{e^{0{\cdot}1t}}{e^{-0{\cdot}05t}} = e^{0{\cdot}15t}$$

$$\ln \frac{39}{11} = 0{\cdot}15t$$

$$8{\cdot}438 = t$$

$$\therefore \boxed{\text{During } 2018}$$

(f) Find the predicted average population in Avalon from the beginning of 2010 to the beginning of 2025.

$$\text{Ave} = \frac{1}{15-0}\int_0^{15} 3{\cdot}9e^{-0{\cdot}05t} \times 10^6$$

$$= 260000\left[\frac{e^{-0{\cdot}05t}}{-0{\cdot}05}\right]_0^{15}$$

$$= \frac{260000}{-0{\cdot}05}\left(e^{-0{\cdot}05(15)} - e^{-0{\cdot}05(0)}\right)$$

$$= \boxed{2,743,694}$$

(g) Use the function $q(t) = 3{\cdot}9e^{-0{\cdot}05t} \times 10^6$ to find the predicted rate of change of the population in Avalon at the beginning of 2018.

$$\frac{dq}{dt} = -0.05\left(3.9e^{-0.05t}\right)\left(10^6\right)$$

$$t = 8 \Rightarrow \frac{dq}{dt} = -0.05\left(3.9e^{-0.05(8)}\right) \times 10^6$$

$$= \boxed{-130712}$$

SEC Set B
2017 P1