Question 3
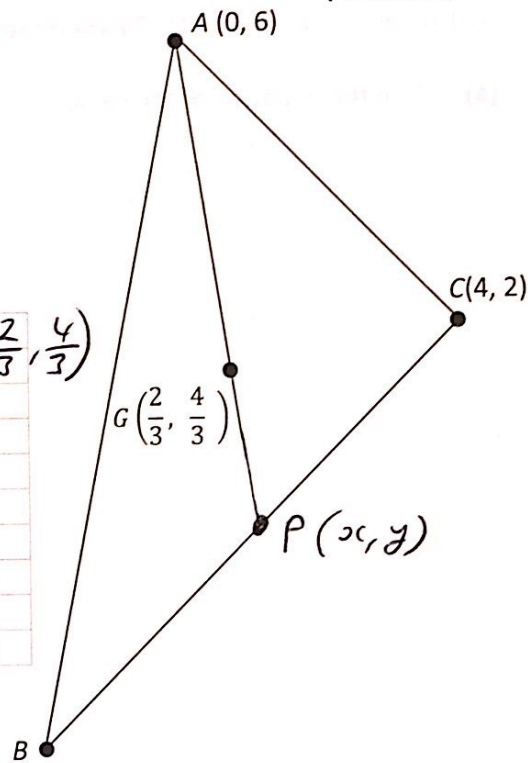
ABC is a triangle where the co-ordinates of A and C are
(0, 6) and (4, 2) respectively.

$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle ABC.

AG intersects BC at the point P.

$|AG| : |GP| = 2 : 1$.

A (0, 6)

C(4, 2)

$G\left(\frac{2}{3}, \frac{4}{3}\right)$

$P(x, y)$

B

(a) Find the co-ordinates of P.

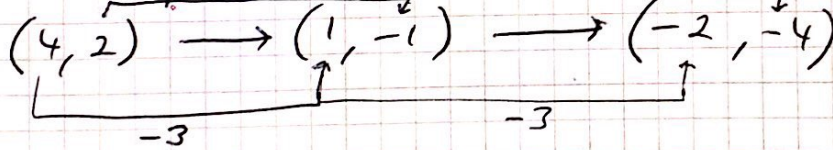$$\left(\frac{2(x) + 1(0)}{2+1}, \frac{2(y) + 1(6)}{2+1}\right) = \left(\frac{2}{3}, \frac{4}{3}\right)$$

$$\therefore \quad \frac{2x}{3} = \frac{2}{3} \qquad \boxed{x = 1}$$

$$\frac{2y + 6}{3} = \frac{4}{3}$$

$$2y = -2$$

$$\boxed{y = -1} \qquad P = \boxed{(1, -1)}$$

(b) Find the co-ordinates of B.

P is midpoint of BC

$$\underset{-3}{\overset{-3}{(4, 2)}} \longrightarrow \underset{}{\overset{-3}{(1, -1)}} \longrightarrow (-2, -4)$$

$$B = \boxed{(-2, -4)}$$

(c) Prove that C is the orthocentre of the triangle ABC.

$$\text{Slope } AC = \frac{2 - 6}{4 - 0} = \frac{-4}{4} = -1$$

$$\text{Slope } BC = \frac{-4 - 2}{-2 - 4} = \frac{-6}{-6} = 1$$

$$(-1)(1) = -1 \qquad \therefore \quad AC \perp BC$$

$$\therefore \quad C \text{ is orthocentre}$$