Question 4 **(25 marks)**

$A(0, 0)$, $B(6{\cdot}5, 0)$ and $C(10, 7)$ are three points on a circle.

(a) Find the equation of the circle.

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(0,0) \Rightarrow 0+0+0+0+c=0 \Rightarrow \boxed{c=0}$

$(6.5,0) \Rightarrow 42.25 + 0 + 13g + 0 + 0 = 0$

$\therefore \boxed{g = -3{\cdot}25}$

$(10,7) \Rightarrow 100 + 49 + 2(-3{\cdot}25)(10) + 2f(7) + 0 = 0$

$\therefore 84 + 14f = 0$

$\boxed{f = -6}$

$\therefore \boxed{x^2 + y^2 - 6{\cdot}5x - 12y = 0}$

(b) Find $|\angle BCA|$. Give your answer in degrees, correct to 2 decimal places.

$\text{Slope } BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-0}{10-6.5} = 2$

$\text{Slope } CA = \frac{0-7}{0-10} = \frac{7}{10}$

$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$

$= \pm\frac{2 - \frac{7}{10}}{1 + (2)(\frac{7}{10})} = \pm\frac{13}{24}$

$\therefore \theta = \tan^{-1}\frac{13}{24} = \boxed{28{\cdot}44^\circ}$

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