Question 8 **(60 marks)**

(a) In 2015, in a particular country, the weights of 15 year olds were normally distributed with a mean of 63·5 kg and a standard deviation of 10 kg.

(i) In 2015, Mariska was a 15 year old in that country. Her weight was 50 kg.
Find the percentage of 15 year olds in that country who weighed more than Mariska.

$\mu = 63.5 \qquad \sigma = 10$

$$z = \frac{x - \mu}{\sigma} = \frac{50 - 63.5}{10} = -1.35$$

$$P(z > -1.35) = P(z < 1.35) = 0.9115$$

$\therefore$ 91.15%

(ii) In 2015, Kamal was a 15 year old in that country.
1·5% of 15 year olds in that country were heavier than Kamal.
Find Kamal's weight.

$P(k > z) = 0.015$

$\therefore P(k < z) = 0.985$

$\therefore z = 2.17$

$\frac{x - 63.5}{10} = 2.17$

$\therefore$ **$x = 85.2$ kg**

(iii) In 2016, 150 of the 15 year olds in that country were randomly selected and their weights recorded. It was found that their weights were normally distributed with a mean weight of 62 kg and a standard deviation of 10 kg. Test the hypothesis, at the 5% level of significance, that the mean weight of 15 year olds, in that country, had not changed from 2015 to 2016. State the null hypothesis and your alternative hypothesis. Give your conclusion in the context of the question.

H_0 = mean weight has <u>not</u> changed

H_A = mean weight <u>has</u> changed

$n = 150 \qquad \bar{x} = 62 \qquad \sigma = 10$

Test statistic $= z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$= \frac{62 - 63.5}{\frac{10}{\sqrt{150}}}$

$= -1.8371 > -1.96$

$\therefore$ Don't reject H_0

$\therefore$ Mean weight has <u>not</u> changed.

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(b) In Galway, rain falls in the morning on $\frac{1}{3}$ of the school days in the year.

When it is raining the probability of heavy traffic is $\frac{1}{2}$.

When it is not raining the probability of heavy traffic is $\frac{1}{4}$.

When it is raining and there is heavy traffic, the probability of being late for school is $\frac{1}{2}$.

When it is not raining and there is no heavy traffic, the probability of being late for school is $\frac{1}{8}$.

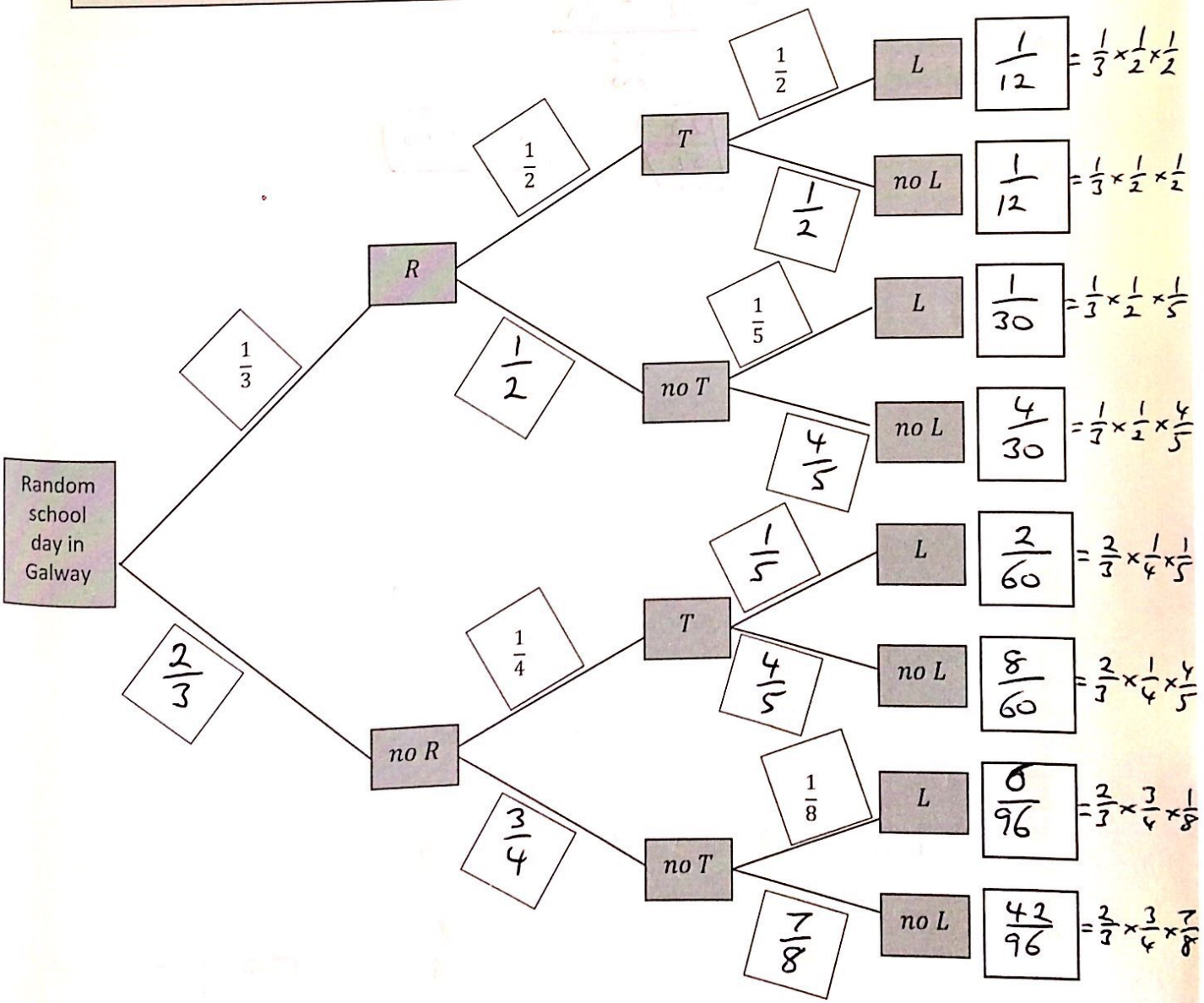
In any other situation the probability of being late for school is $\frac{1}{5}$.

Some of this information is shown in the tree diagram below.

(i) Write the probability associated with each branch of the tree diagram **and** the probability of each outcome into the blank boxes provided.

Give each answer in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.

Key	Rain $= R$	Heavy traffic $= T$	Late $= L$
	No rain $= no\ R$	Not heavy traffic $= no\ T$	Not late $= no\ L$.



(ii) On a random school day in Galway, find the probability of being late for school.

$$P(\text{late}) = \frac{1}{12} + \frac{1}{30} + \frac{2}{60} + \frac{6}{96}$$

$$= \boxed{\frac{17}{80}} \quad = \boxed{0.2125}$$

(iii) On a random school day in Galway, find the probability that it rained in the morning, given that you were late for school.

$$P(R|L) = \frac{P(R \cap L)}{P(L)}$$

$$= \frac{\frac{1}{12} + \frac{1}{30}}{\frac{17}{80}}$$

$$= \boxed{\frac{28}{51}} \quad = \boxed{0.5490}$$

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