

Question 9

(50 marks)

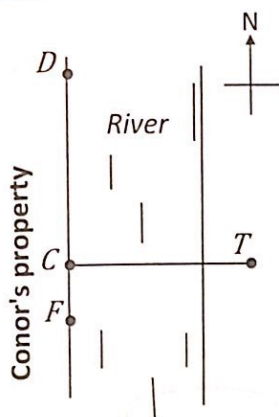


Figure 1

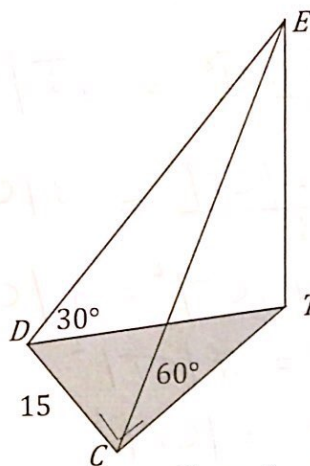


Figure 2

SEC Set B  
2017 P2

Conor's property is bounded by the straight bank of a river, as shown in **Figure 1** above.  $T$  is the base of a vertical tree that is growing near the opposite bank of the river.  $|TE|$  is the height of the tree, as shown in **Figure 2** above. From the point  $C$ , which is due west of the tree, the angle of elevation of  $E$ , the top of the tree, is  $60^\circ$ . From the point  $D$ , which is 15 m due north of  $C$ , the angle of elevation of  $E$  is  $30^\circ$  (see **Figure 2**). The land on both sides of the river is flat and at the same level.

(a) Use triangle  $ECT$ , to express  $|TE|$  in the form  $\sqrt{a}|CT|$  metres, where  $a \in \mathbb{N}$ .

$\tan 60^\circ = \frac{x}{y}$   
 $\therefore \sqrt{3} = \frac{x}{y}$   
 $\therefore \boxed{\sqrt{3}|CT| = |TE|}$

(b) Show that  $|TE|$  may also be expressed as  $\sqrt{\frac{225+|CT|^2}{3}}$  metres.

$\tan 30^\circ = \frac{x}{z} \quad \therefore \frac{1}{\sqrt{3}}z = x \quad \text{--- (1)}$   
 $\therefore \frac{1}{\sqrt{3}}|DT| = |TE|$   
 $z^2 = 15^2 + y^2$   
 $\therefore z = \sqrt{225 + y^2}$   
 Sub in (1)  
 $\frac{\sqrt{225 + y^2}}{\sqrt{3}} = x \quad \therefore \boxed{\sqrt{\frac{225 + |CT|^2}{3}} = |TE|}$

- (c) Hence find  $|CT|$ , the distance from the base of the tree to the bank of the river at Conor's side. Give your answer correct to 1 decimal places.

$$\sqrt{\frac{225 + |CT|^2}{3}} = \sqrt{3} |CT|$$

$$\therefore \frac{225 + |CT|^2}{3} = 3 |CT|^2$$

$$\therefore 225 + |CT|^2 = 9 |CT|^2$$

$$\therefore 225 = 8 |CT|^2$$

$$\frac{225}{8} = |CT|^2 \quad \therefore |CT| = \boxed{5.3 \text{ m}}$$

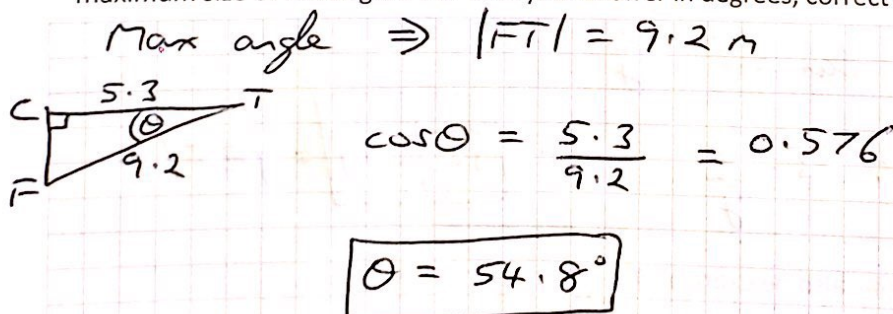
- (d) Find  $|TE|$ , the height of the tree. Give your answer correct to 1 decimal place.

$$|TE| = \sqrt{3} |CT|$$

$$= \sqrt{3} (5.3)$$

$$= \boxed{9.2 \text{ m}}$$

- (e) The tree falls across the river and hits the bank at Conor's side at the point  $F$ . Find the maximum size of the angle  $FTC$ . Give your answer in degrees, correct to 1 decimal place.



- (f) If the tree was equally likely to fall in any direction, find the probability that it would hit the bank at Conor's side, when it falls. Give your answer as a percentage, correct to 1 decimal place.

$$\text{Probability} = \frac{\text{No. of ways (angles) we want}}{\text{Total no. of possible ways (angles)}}$$

$$= \frac{(54.8)(2)}{360}$$

$$= 0.304$$

$$\Rightarrow \boxed{30.4\%}$$

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