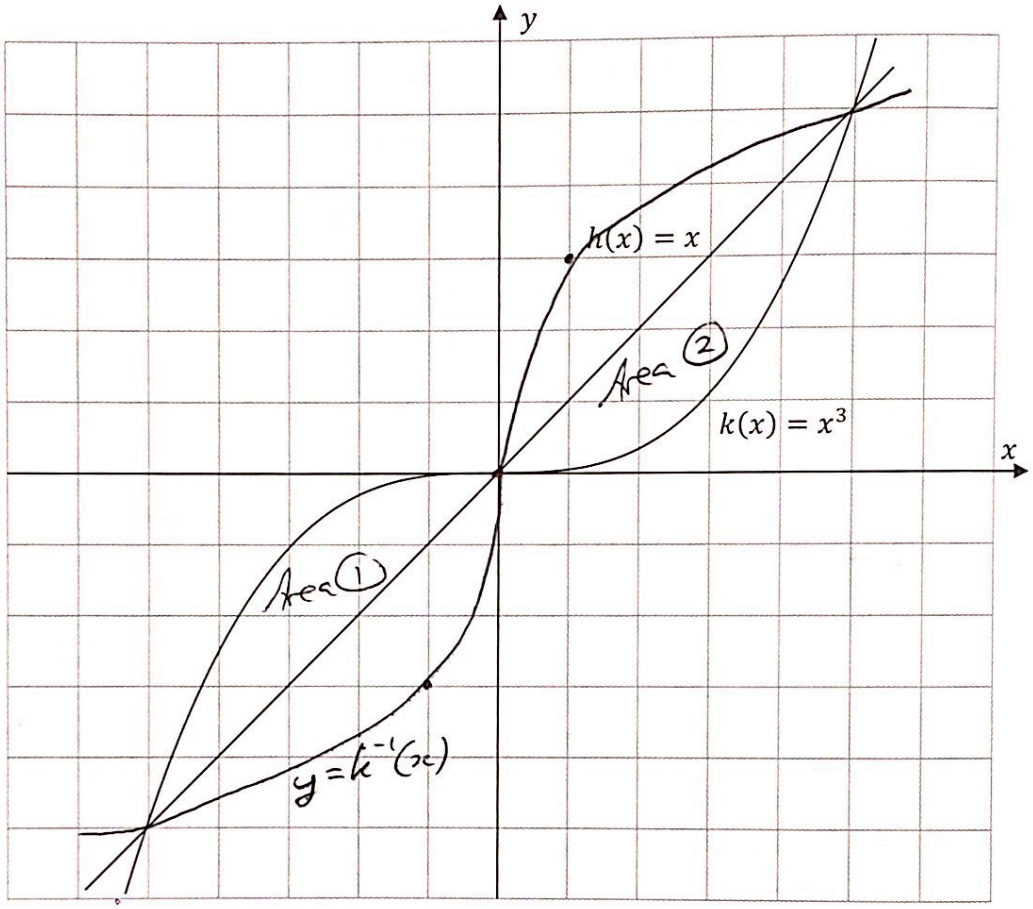
Question 6 **(25 marks)**

Parts of the graphs of the functions $h(x) = x$ and $k(x) = x^3,\ x \in \mathbb{R}$, are shown in the diagram below.



(a) Find the co-ordinates of the points of intersection of the graphs of the two functions.

$x = x^3$

$\therefore\ x^3 - x = 0$

$\therefore\ x(x^2 - 1) = 0$

$\therefore\ x(x-1)(x+1) = 0$

$x = 0$ or $x = 1$ or $x = -1$

$y = 0$ $\quad y = 1$ $\quad y = -1$

$(0,0)$ $\quad (1,1)$ $\quad (-1,-1)$

(b) **(i)** Find the total area enclosed between the graphs of the two functions.

$$\text{Area } ① = \text{Area } ② = \int_{-1}^{0} (x^3 - x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0}$$

$$= 0 - \left(\frac{(-1)^4}{4} - \frac{(-1)^2}{2}\right)$$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4}$$

$\therefore$ Total area $= 2\left(\frac{1}{4}\right) = \boxed{\frac{1}{2}}$ units^2

(ii) On the diagram on the previous page, using symmetry or otherwise, draw the graph of k^{-1}, the inverse function of k.

see graph