Question 3 **(25 marks)**

(a) A security code consists of six digits chosen at random from the digits 0 to 9.
The code may begin with zero and digits may be repeated.
For example

0	7	1	7	3	7

is a valid code.

(i) Find how many of the possible codes will end with a zero.

$10 \times 10 \times 10 \times 10 \times 10 \times 1$

$= 100,000$

(ii) Find how many of the possible codes will contain the digits 2 0 1 8 together and in this order.

Stick 2018 together $\Rightarrow$ 3 places

1 10 10 $= 100$

$\times \quad 3 \leftarrow$ position of 2018 in the code

300

(b) Find a, b, c, and d, if $\dfrac{(n+3)!\,(n+2)!}{(n+1)!(n+1)!} = an^3 + bn^2 + cn + d$, where a, b, c, and $d \in \mathbb{N}$.

$$\frac{(n+3)(n+2)\cancel{(n+1)!}\,(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}\;\cancel{(n+1)!}} = an^3+bn^2+cn+d$$

$$\therefore (n^2+5n+6)(n+2) = an^3+bn^2+cn+d$$

$$\therefore n^3+5n^2+2n^2+6n+10n+12 = an^3+bn^2+cn+d$$

$\therefore$ By comparing coefficients

$a = 1$	$b = 7$	$c = 16$	$d = 12$