

Question 5

(25 marks)

The line  $m: 2x + 3y + 1 = 0$  is parallel to the line  $n: 2x + 3y - 51 = 0$ .

(a) Verify that  $A(-2, 1)$  is on  $m$ .

$$2(-2) + 3(1) + 1$$

$$= -4 + 3 + 1 = 0 \quad \therefore A \text{ on } m$$

(b) Find the coordinates of  $B$ , the point on the line  $n$  closest to  $A$ , as shown below.

line  $AB \perp m$

$\therefore$  eqn  $AB =$

$$3x - 2y + k = 0$$

$(-2, 1) \Rightarrow 3(-2) - 2(1) + k = 0$

$$k = 8$$

$\therefore 3x - 2y + 8 = 0$

$n \Rightarrow 2x + 3y - 51 = 0$

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$$9x - 6y + 24 = 0$$

$$4x + 6y - 102 = 0$$

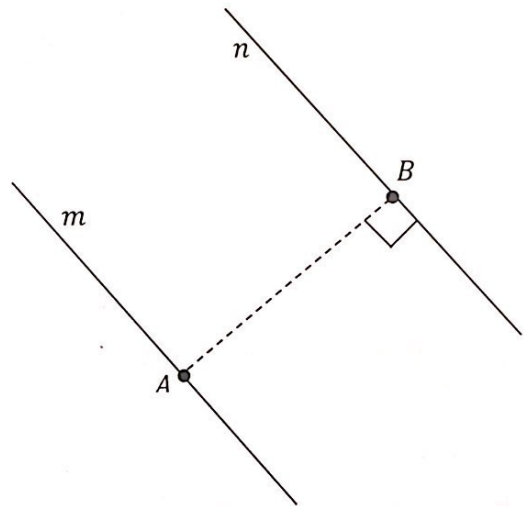

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$$13x - 78 = 0$$

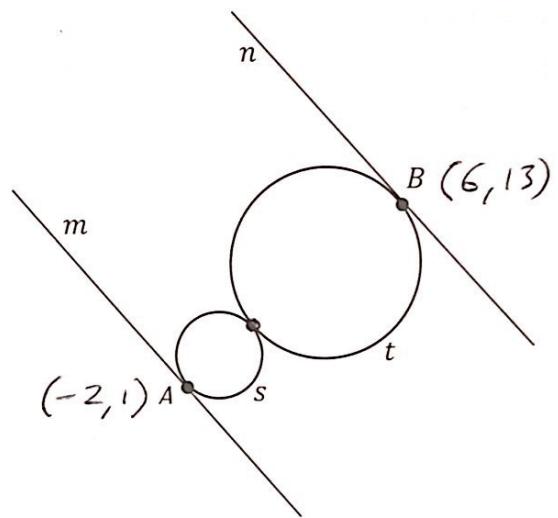
$$x = 6$$

$$y = \frac{-8 - 3(6)}{-2} = 13$$

$(6, 13)$



- (c) Two touching circles,  $s$  and  $t$ , are shown in the diagram.  $m$  is a tangent to  $s$  at  $A$  and  $n$  is a tangent to  $t$  at  $B$ . The ratio of the radius of  $s$  to the radius of  $t$  is  $1 : 3$ . Find the equation of  $s$ .



To find centre of  $s$

Divide  $AB$  in ratio  $1:7$

$$\left( \frac{1(6) + 7(-2)}{1+7}, \frac{1(13) + 7(1)}{1+7} \right)$$

$$= \left( \frac{-8}{8}, \frac{20}{8} \right)$$

$$= (-1, 2.5)$$

$$\text{Radius} = \sqrt{(-2+1)^2 + (1-2.5)^2} = \sqrt{3.25}$$

$$\text{Eqn } s = (x+1)^2 + (y-2.5)^2 = 3.25$$