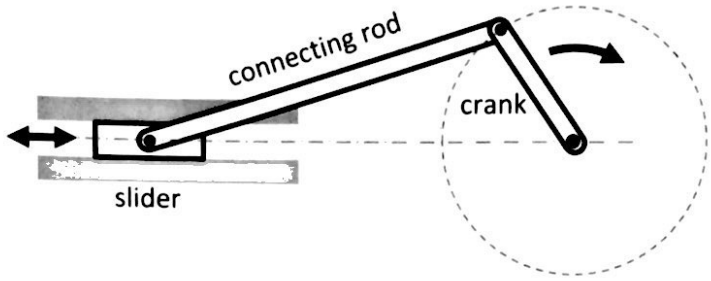
Question 9 **(40 marks)**

In engineering, a crank-and-slider mechanism can be used to change circular motion into motion back and forth in a straight line.



In the diagrams below, the crank $[OD]$ rotates about the fixed point O. The point C slides back and forth in a horizontal line. $[CD]$ is the rod that connects C to the crank. The diagrams below show three of the possible positions for C and D. $|OD| = 10$ cm and $|DC| = 30$ cm.

Diagram 1
(Starting position)

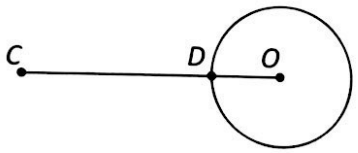


Diagram 2

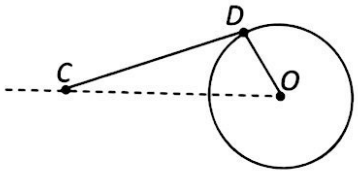
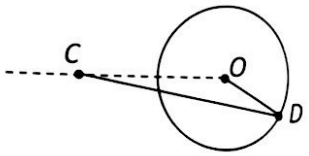
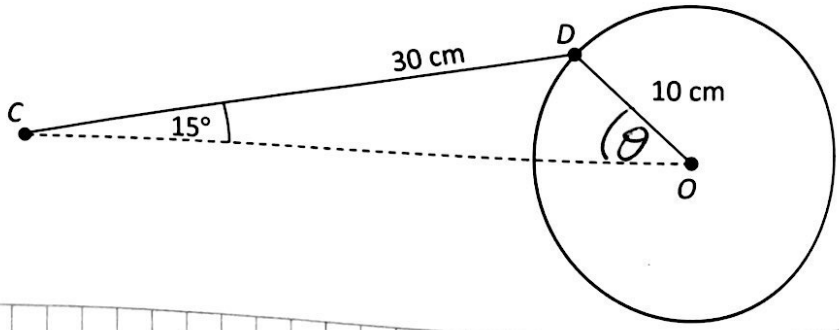


Diagram 3



(a) The diagram below shows a particular position of the mechanism with $|\angle DCO| = 15°$. Find $|\angle COD|$, correct to the nearest degree.



$$\frac{\sin\theta}{30} = \frac{\sin 15^\circ}{10}$$

$$\sin\theta = 0.776$$

$$\theta = 51^\circ$$

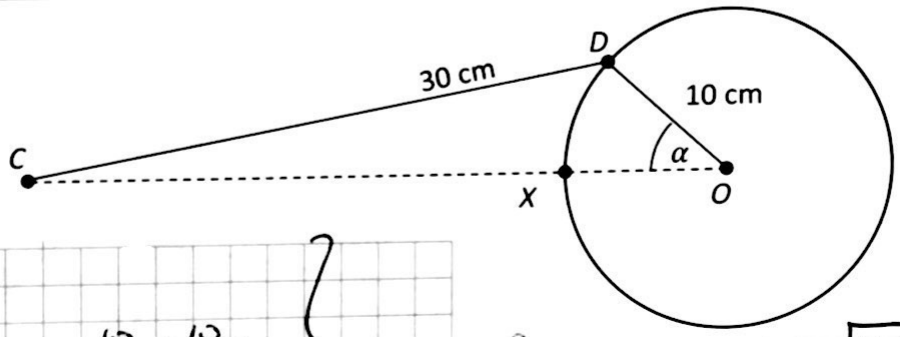
(b) As D moves in a circle around O, the angle α in the diagram below increases. The distance $|CX|$ can be considered to be a function of α and written as $f(\alpha)$.

(i) Write down the period and range of f.

Period = 360°	Range = $[10, 30]$

(ii) Complete the table below for $f(\alpha)$.
Give your answers correct to 2 decimal places where appropriate.
(**Note: Diagram 1** at the start of this question represents $\alpha = 0°$).

α	$0°$	$90°$	$180°$	$270°$	$360°$
$f(\alpha)$ (cm)	30	18·28	10	18·28	30



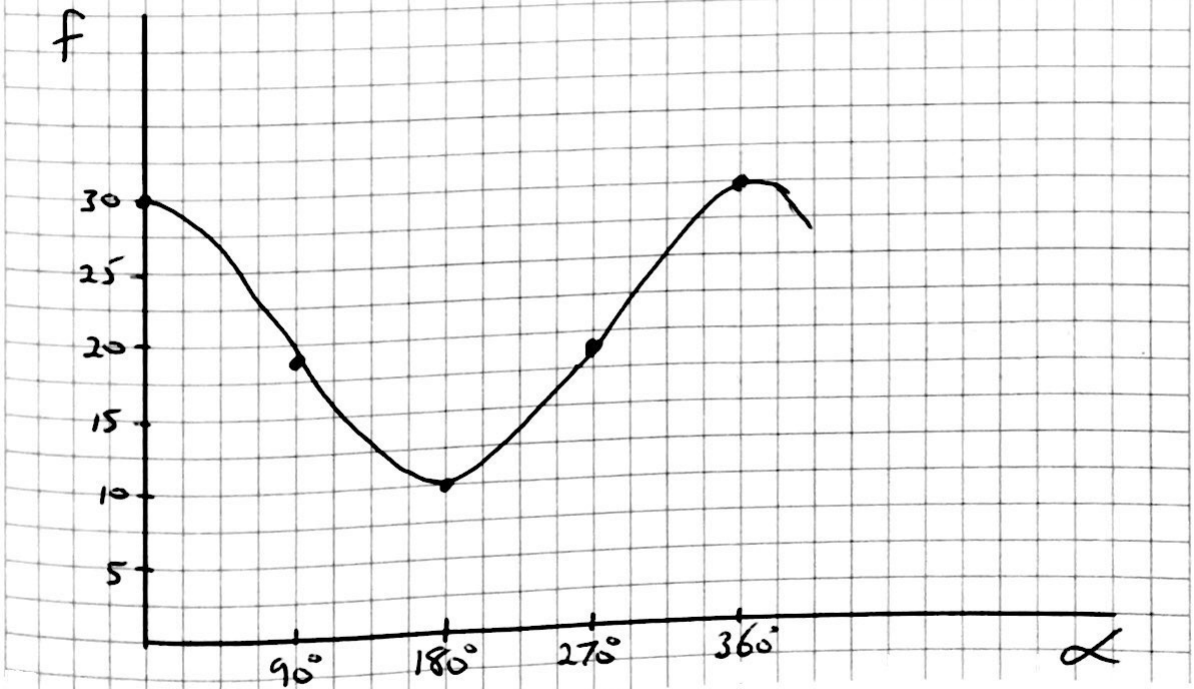
$\alpha = 180°$
$\Rightarrow$
C — 10 — X — 10 — O — 10 — D

$\alpha = 90°$
$\Rightarrow$
C — 30 — D, 18·28, O, 10

$x = \sqrt{30^2 - 10^2}$
$= 28{\cdot}28$

(iii) Use your values from the table to draw a rough sketch of f in the domain $0° \leq \alpha \leq 360°$.

Draw your graph on this grid.



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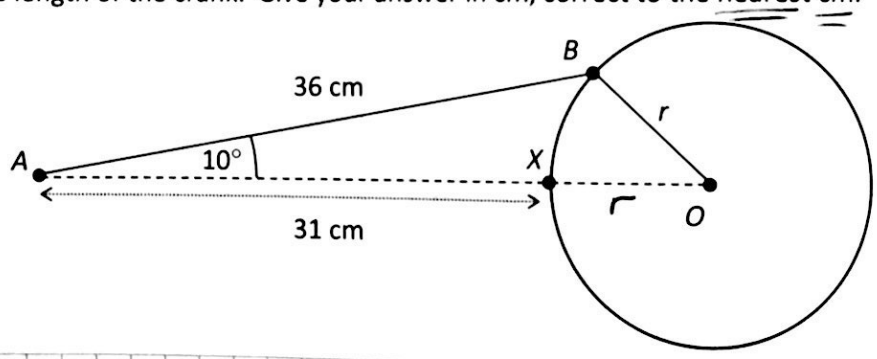
(iv) Referring to **Diagrams 1, 2, and 3** near the start of this question, for which of the three positions of the mechanism will a 1 degree change in α cause the greatest change in the position of C? Explain your answer.

Answer: Diagram 2

Explanation:

Looking at the graph in part (iii) it is steeper when α is close to 90° than when it is close to 0° or 180°.

(c) The diagram below shows another crank-and-slider mechanism with different dimensions. In the diagram, $|AB| = 36$ cm, $|AX| = 31$ cm, and $|\angle BAO| = 10°$.
(**Note**: $|\angle OBA| \neq 90°$)
Find r, the length of the crank. Give your answer in cm, correct to the nearest cm.



$r^2 = 36^2 + (31+r)^2 - 2(36)(31+r)\cos 10°$

$\therefore r^2 = 1296 + 961 + 62r + r^2 - 2232\cos 10° - 72r\cos 10°$

$\therefore 8.906r = 58.91$

$\therefore r = 6.62$

$\therefore \boxed{r \approx 7 \text{ cm}}$