

ALGEBRA HW 3 SOLUTIONS

- ① (i) NO REAL ROOTS (f)
TWO SAME, REAL ROOTS (g)
TWO DIFFERENT, REAL ROOTS (h)

(ii) EQUAL ROOTS $\Rightarrow b^2 - 4ac = 0$

$$\therefore (1-k)^2 - 4(k)(k) = 0$$

$$\therefore 1 - 2k + k^2 - 4k^2 = 0$$

$$\therefore 1 - 2k - 3k^2 = 0$$

$$\therefore 0 = 3k + 2k - 1$$

$$\therefore 0 = (3k - 1)(k + 1)$$

$$\boxed{k = \frac{1}{3}} \text{ or } \boxed{k = -1}$$

②
$$\frac{x^2 + 4}{(x-2)(x+2)} - \frac{x}{x+2} = \frac{x^2 + 4 - x(x-2)}{(x-2)(x+2)}$$

$$= \frac{x^2 + 4 - x^2 + 2x}{(x-2)(x+2)} = \frac{2x + 4}{(x-2)(x+2)} = \frac{2(x+2)}{(x-2)(x+2)}$$
$$= \boxed{\frac{2}{x-2}}$$

③ $(x-p)(x+p) = x^2 - p^2$

$$\therefore (x^2 - p^2)\left(ax - \frac{d}{p^2}\right) = ax^3 + bx^2 + cx + d$$
$$\therefore ax^3 - \frac{d}{p^2}x^2 - p^2ax + d = ax^3 + bx^2 + cx + d$$

$$\left. \begin{array}{l} \frac{x^2 \text{ coeff}}{-\frac{d}{p^2}} = b \\ -ap^2 = c \end{array} \right\} \begin{array}{l} \frac{x \text{ coeff}}{-ap^2} = c \\ \Rightarrow bc = \left(-\frac{d}{p^2}\right)(-ap^2) \\ = ad \end{array}$$