

2004

⑥ b

$$x = a \cos(\omega t - \beta)$$

(i). $v = \frac{dx}{dt} = a(-\sin(\omega t - \beta)) \cdot \omega$

$$v = -a\omega \sin(\omega t - \beta)$$

$$a = \frac{d^2x}{dt^2} = -a\omega \cos(\omega t - \beta) \cdot \omega$$

$$a = -\omega^2 a \cos(\omega t - \beta)$$

$$\boxed{a = -\omega^2 x} \quad \therefore \underline{\underline{\text{SHM}}}$$

(ii). $T = 16 \text{ s}$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8}$$

$$\therefore \boxed{\omega = \frac{\pi}{8} \text{ rad/s}}$$

@ $t = 4$, $x = 12$ $\therefore 12 = a \cos\left(\frac{4\pi}{8} - \beta\right)$

A ~~(*)~~ $12 = a \cos\left(\frac{\pi}{2} - \beta\right)$

@ $t = 8$, $x = -5$ $\therefore -5 = a \cos\left(\frac{8\pi}{8} - \beta\right)$

B ~~(*)~~ $-5 = a \cos(\pi - \beta)$

• from tables $\Rightarrow \cos(A-B) = \cos A \cos B + \sin A \sin B$.

$$\textcircled{A}: 12 = a \left[\cos \frac{\pi}{2} \cos \beta + \sin \frac{\pi}{2} \sin \beta \right]$$

$$12 = a \left[0 + \sin \beta \right]$$

$$\boxed{12 = a \sin \beta}$$

$$\textcircled{B} -5 = a \left[\cos \pi \cos \beta + \sin \pi \sin \beta \right]$$

$$-5 = a \left[-\cos \beta + 0 \right]$$

$$-5 = -a \cos \beta$$

$$5 = a \cos \beta$$

$$\boxed{a = \frac{5}{\cos \beta}}$$

• Put (substitute) \textcircled{B} into \textcircled{A} :

$$12 = \left(\frac{5}{\cos \beta} \right) \sin \beta$$

$$\frac{\sin \beta}{\cos \beta} = \frac{12}{5}$$

$$\tan \beta = \frac{12}{5}$$

$$\text{Ans} = \tan^{-1} \left(\frac{12}{5} \right) = \boxed{\beta = 1.176} \text{ rads}$$

$$\boxed{\begin{aligned} a &= \frac{5}{\cos \beta} \\ a &= \frac{5}{\cos(1.176)} \\ \boxed{a} &= 13 \end{aligned}}$$