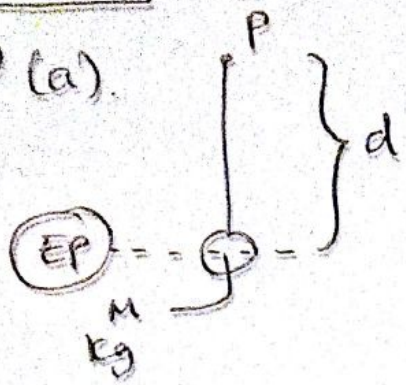
2007

(6) (a).



(i). particle at equilibrium when $F_{up} = F_{down}$.

- $F_{up} = -kd$.
- $F_{down} = Mg$

$\therefore \quad Mg = kd.$ (A)

- Let's extend a further distance x, and measure resultant force on particle.

$$F = F_{down} - F_{up}.$$

- $F_{down} = Mg$
- $F_{up} = k(d + x)$.

$\therefore \quad F = Mg - kd - kx.$

- but: $kd = Mg$! (from (A))

$\therefore \quad F = Mg - Mg - kx.$

$$F = -kx.$$

$$Ma = -kx.$$

$$\boxed{a = -\frac{k}{M}x}$$

$\therefore$ SHM

also...

$$\omega^2 = \frac{k}{M}.$$

$$\omega = \sqrt{\frac{k}{M}}$$

(ii). $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/M}} = 2\pi\sqrt{\frac{M}{k}}$.

- but: $k = \frac{Mg}{d}$ (from (A)).

$\therefore T = 2\pi\sqrt{\frac{M}{\left(\frac{Mg}{d}\right)}} = 2\pi\sqrt{M\left(\frac{d}{Mg}\right)}$

$= \boxed{2\pi\sqrt{\frac{d}{g}}}$.