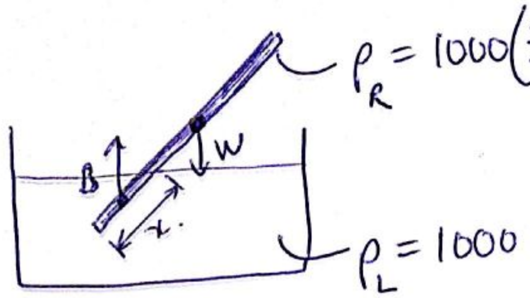
2008

(9) (a).



- B = weight of liquid displaced.

$$B = \rho_L V g = (1000)(x)(A)(g).$$

↑ cross-sectional area

- W = weight of rod

$$W = 2A\rho_R g$$

$$W = 2A\left(\frac{7000}{9}\right)g$$

$$9W = 14000Ag$$

$$A = \frac{9W}{14000g}.$$ sub. into B.

- $B = (1000)(x)\left(\frac{9W}{14000g}\right)(g)$

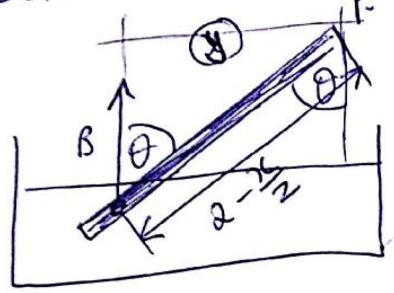
$$B = \frac{9Wx}{14}.$$

Moments

- Pick end of rod (P) as equilibrium position.

@ equilibrium, CW moments = ACW moments.

- B (clockwise).

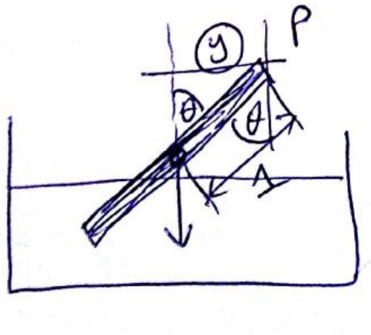


(CW) Moment = force × perp.-distance.

$$= \frac{9Wx}{14}\left(2 - \frac{x}{2}\right)\sin\theta.$$

(y). (in diagram).

- W (anti-clockwise).



(ACW) Moment $= (W)(1)(\sin\theta)$

CW moments = ACW moments.

$$\frac{9Wx}{14}\left(2-\frac{x}{2}\right)\sin\theta = W\sin\theta$$

$$\therefore \frac{9x}{14}\left(2-\frac{x}{2}\right) = 1.$$

$$9x\left(2-\frac{x}{2}\right) = 14.$$

$$18x - \frac{9x^2}{2} = 14.$$

$$36x - 9x^2 = 28.$$

$$9x^2 - 36x + 28 = 0.$$

$a = 9$
$b = -36$
$c = 28$

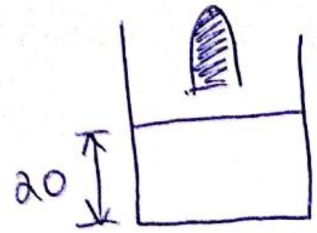
$$x = \frac{36 \pm \sqrt{(-36)^2 - 4(9)(28)}}{2(9)}.$$

$$x = \frac{36 \pm 12\sqrt{2}}{18}.$$

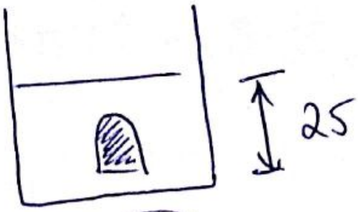
$x = \boxed{1.06}$ or 2.94.

(X) discard — rod length only 2m!

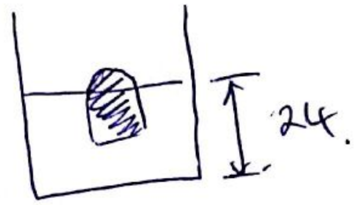
(b).



(before)



(after)



(float)

- volume of liquid (before) $= (0.2)(A)$. ← cross-sectional area of cylinder.

$$= \underline{\underline{0.2\pi r^2}}$$

(float) $= V_I + 0.2\pi r^2 = 0.24\pi r^2$.

V_I: volume of immersed part of object.

$$\therefore V_I = \underline{\underline{0.04\pi r^2}}.$$

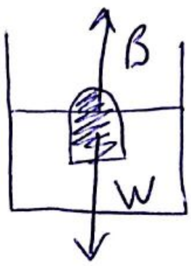
(after) $= V_0 + 0.2\pi r^2 = 0.25\pi r^2$.

$V_0 = 0.05\pi r^2$.

volume of total object

- when object floating

i.e. $B = W$.



$B = (1000)(0.04\pi r^2)(g)$

$W = \rho_0 (0.05\pi r^2)(g)$.

$\therefore (1000)(0.04\pi r^2)(g) = \rho_0 (0.05\pi r^2)(g)$.

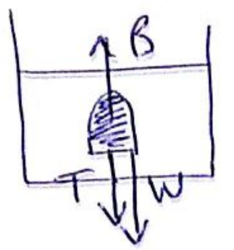
$0.05\rho_0 = 40$

$\rho_0 = \frac{40}{0.05} = 800$

rel. density $s = \frac{800}{1000} = \boxed{0.8}$

(ii)

- when object tied to bottom:



$$F_{up} = F_{down}$$

$$B = T + W.$$

$$B = T + (0.06)g$$

$$T = B - 0.06g$$

$$T = (1000)(\text{volume})(g) - 0.06g.$$

$$\uparrow$$

$$\rho_o = \frac{M_o}{V_o}.$$

$$\therefore V_o = \frac{m_o}{\rho_o} = \frac{0.06}{800}$$

$$= 7.5 \times 10^{-5}.$$

$$T = (1000)(7.5 \times 10^{-5})(g) - 0.06g$$

$$T = 0.075g - 0.06g$$

$$T = \boxed{0.015g\ N}$$

$$= 0.147 N.$$

(iii).

$$V_0 = 0.05\pi r^2.$$

$$7.5\times10^{-5} = 0.05\pi r^2.$$

$$r^2 = \frac{7.5\times10^{-5}}{0.05\pi}.$$

$$r = 0.02185\ m$$

$$= \boxed{2.19\,cm}$$