$\boxed{2012}$

6 (a) $m = 0.5$ kg

$A = 0.2$ m

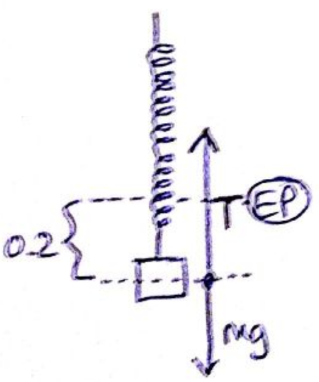
$T = 2$ s

(i) $a_{max} = \omega^2 A$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi.$$

$$\therefore a_{max} = \pi^2(0.2) = \boxed{\frac{\pi^2}{5}}$$

(ii). max. force exerted by the spring



- resultant force at amplitude

$$F = ma_{max}$$
$$= \left(\frac{1}{2}\right)\left(\frac{\pi^2}{5}\right) = \frac{\pi^2}{10}.$$

- balance of two forces - up and down.

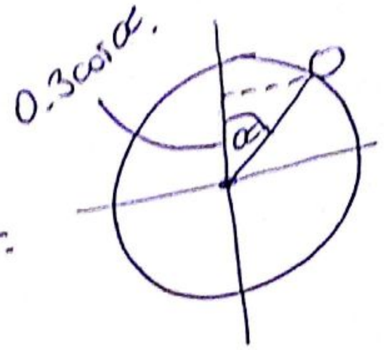
$$\frac{\pi^2}{10} = T - mg$$

$$T = \frac{\pi^2}{10} + \frac{9}{2}$$

$$= \boxed{5.9N}$$

(b). PE → KE $\boxed{\text{* energy.}}$

BEFORE AFTER
PE + KE = PE + KE



$mgh + \frac{1}{2}mv^2 = mgh + \frac{1}{2}mv^2.$
(before) (after)

$0.3mg + 0 = (0.3\cos\alpha)mg + \frac{1}{2}mv^2.$

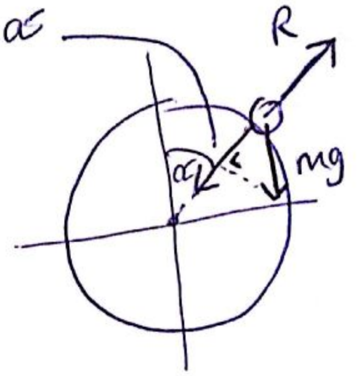
$\frac{1}{2}mv^2 = 0.3mg - 0.3\cos\alpha\, mg.$

$\frac{1}{2}mv^2 = 0.3mg(1 - \cos\alpha)$

$\cancel{m}v^2 = 0.6\cancel{m}g(1 - \cos\alpha)$

A • $v^2 = 0.6g(1 - \cos\alpha).$

$\boxed{\text{* FORCES}}$



centripetal force $= \frac{mv^2}{0.3}$

$mg\cos\alpha - R = \frac{mv^2}{0.3}.$

when particle leaves surface, R = 0.

$$\therefore \cancel{M}g\cos\alpha = \frac{\cancel{M}v^2}{0.3}$$

(B) $v^2 = 0.3g\cos\alpha.$

Bring (A) and (B) together:

$$0.6\cancel{g}(1-\cos\alpha) = 0.3\cancel{g}\cos\alpha.$$

$$2(1-\cos\alpha) = \cos\alpha.$$

$$2 - 2\cos\alpha = \cos\alpha.$$

$$3\cos\alpha = 2$$

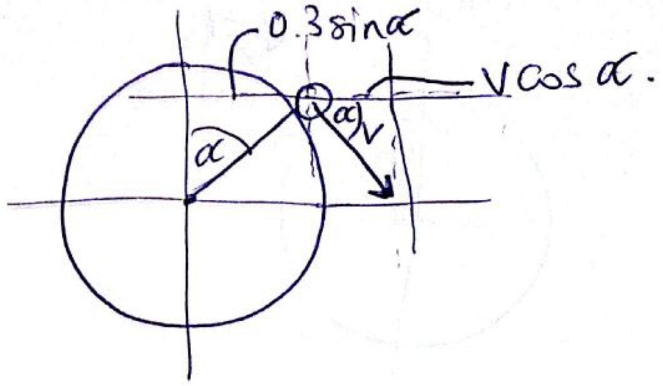
$$\cos\alpha = \frac{2}{3}.$$

Put this angle into (A) or (B).

$$v^2 = 0.3g(2/3).$$

$$v^2 = 0.2g$$

$$v = \boxed{1.4\,\mathrm{m\,s^{-1}}}$$

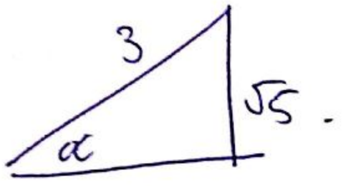


∴ after particle leaves, horiz. distance ⇒

$$x = 0.3\sin\alpha + (1.4)(\cos\alpha)(t).$$

NB.

$\cos\alpha = \frac{2}{3}$



$3^2 = 2^2 + x^2$

$x^2 = 9 - 4$

$x^2 = 5$

$x = \sqrt{5}$

$\therefore \sin\alpha = \frac{\sqrt{5}}{3}.$

$\therefore x = 0.3\left(\frac{\sqrt{5}}{3}\right) + (1.4)\left(\frac{2}{3}\right)t.$

$x = \frac{\sqrt{5}}{10} + \left(\frac{14}{15}\right)t$

$\therefore k = 14/15.$

$k = \boxed{0.93}$