2016

Q6

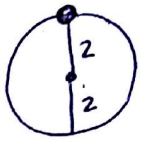
a)

BEFORE	AFTER
KE	KE + PE

(K) Conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh \leftarrow 2r = 4$$

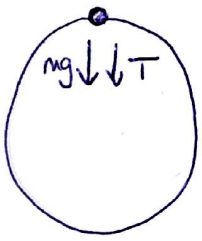
$$\frac{1}{2}u^2 = \frac{1}{2}v^2 + 4g$$



(A) $\boxed{u^2 = v^2 + 8g}$

- at the top:

(K) forces.



Centripetal force = $F_{gravity} + T$

$$\frac{mv^2}{r} = mg + T \leftarrow \text{"least" speed of projection - string is slack at top! } \therefore T = 0$$

$$\frac{mv^2}{2} = mg$$

(B) $\boxed{v^2 = 2g}$

- Combining (A) and (B)

$$2g(2g)^2 = v^2 + 8g$$

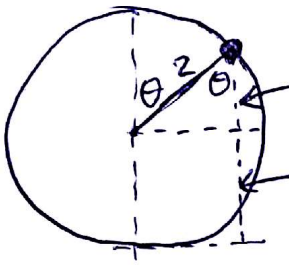
$$4g^2 = v^2 + 8g$$

PTO

$$u^2 = 2g + 8g = 10g$$

$$u = \sqrt{10g} = \sqrt{98} = \boxed{7\sqrt{2}}\ m/s.$$

(ii)



$(PE) = mg(2) + mg(2\cos\theta)$

$= mg(2 + 2\cos\theta).$

(A) conservation of energy

BEFORE	AFTER
(KE)	(KE) + (PE)

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + (PE)$$

$$\frac{1}{2}m(7)^2 = \frac{1}{2}mv^2 + mg(2 + 2\cos\theta)$$

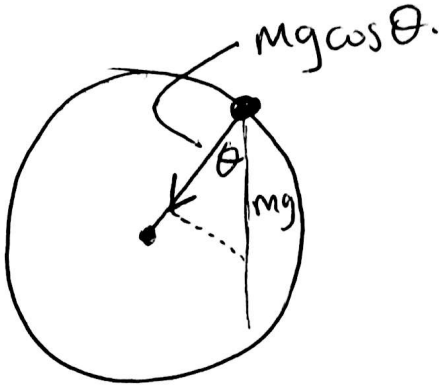
$$49 = v^2 + 2g(2 + 2\cos\theta)$$

$$49 = v^2 + 4g + 4g\cos\theta.$$

$$\therefore v^2 = 49 - 4g - 4g\cos\theta.$$

(A) $v^2 = \boxed{g - 4g\cos\theta}$

(A) forces.



centripetal force = $F_{gravity} + T$

$$\frac{mv^2}{r} = mg\cos\theta + (T) \quad \leftarrow \text{again, slack!}$$

$\therefore T = 0.$

$$\frac{\cancel{m}v^2}{2} = \cancel{m}g\cos\theta$$

(B) $\boxed{v^2 = 2g\cos\theta.}$

- combining (A) and (B).

$$2\cancel{g}\cos\theta = \cancel{g} - 4\cancel{g}\cos\theta.$$

$$2\cos\theta = 1 - 4\cos\theta.$$

$$6\cos\theta = 1$$

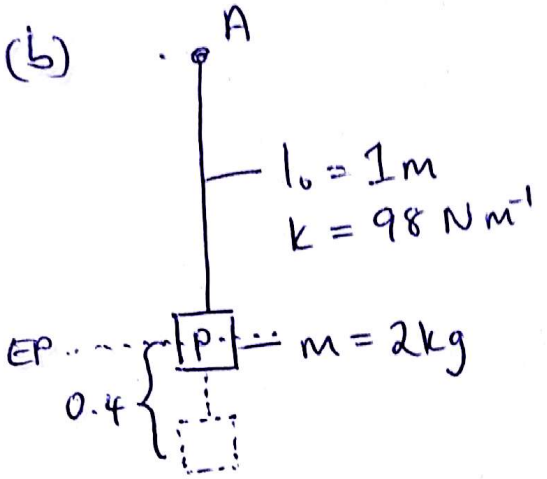
$$\cos\theta = 1/6.$$

$$\theta = \cos^{-1}(1/6) = \boxed{80.41^\circ}$$

| 2016

Q6

(b)



* we need to find EP.

i.e. $F_{up} = F_{down}$

$k(l - l_0) = mg$

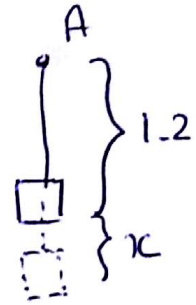
$98(l - 1) = 2(9.8)$

$98l - 98 = 19.6$

$98l = 117.6$

$l = \boxed{1.2m}$

(i) Let's say we pull a distance x from EP.



$F = F_{down} - F_{up}$

$F = mg - k(l - l_0)$

$F = 2(9.8) - 98(1.2 + x - 1)$

$F = 19.6 - 117.6 - 98x + 98$

$F = -98x$

$ma = -98x$

$a = \frac{-98}{2}x$

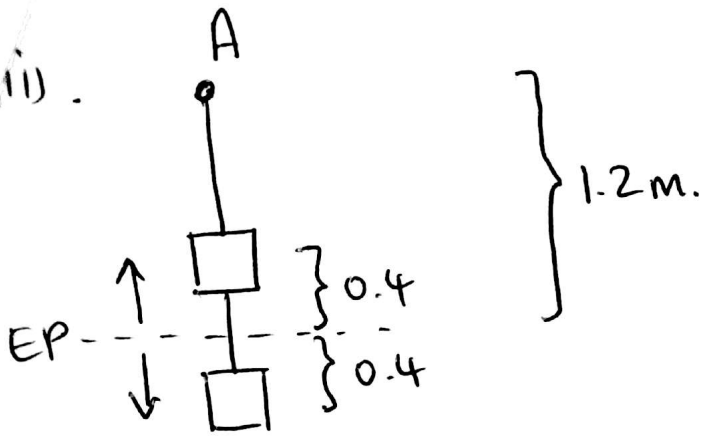
$\omega^2 = 49$

$\boxed{\omega = 7}$ rads s^{-1}

$\boxed{a = -49x}$

$\therefore$ SHM — because a proportional to x and in opposite direction

ii).



* notice that the particle, if SHM would rise 0.4m above EP, i.e. 0.8m below A.
- but lo is 1m!
∴ it will go slack before this point, at $x = +0.2$

- $v^2 = \omega^2(A^2 - x^2)$

$v^2 = (7)^2(0.4^2 - 0.2^2)$

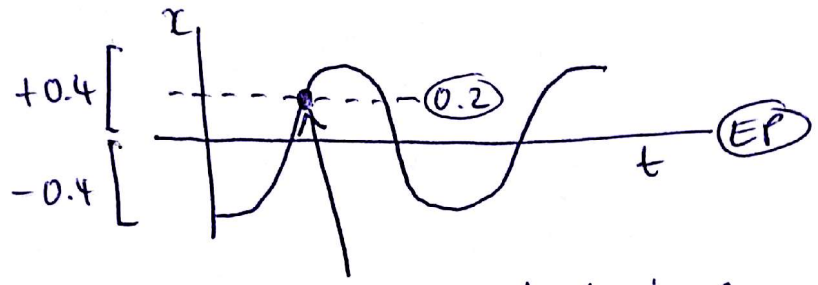
$v^2 = 49(0.12)$

$v^2 = 5.88$

$v = \boxed{2.42 \text{ m s}^{-1}}$

) at max displacement when $t=0$,

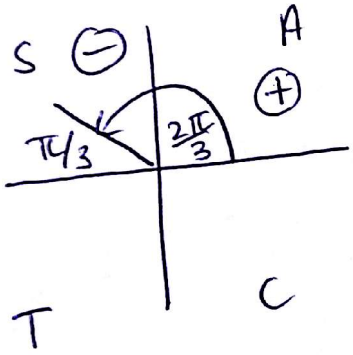
$\therefore x = A\cos\omega t$



when it gets to here, string goes slack!

$\therefore 0.2 = -0.4\cos 7t$

$\cos 7t = -\frac{0.2}{0.4} = -\frac{1}{2}$ (ref angle $= \pi/3$)



$\therefore 7t = \frac{2\pi}{3}$

$t = \frac{2\pi}{21} = \boxed{0.299\,s}$

- From this point, no upwards force!

$\therefore$ ULM

$v = u + at$

$0 = 2.42 - 9.8t$

$9.8t = 2.42$

$t = \boxed{0.247\,s}$

$\therefore$ total $= 0.299 + 0.247 = \boxed{0.546\,s}$