5 MATHS ALGEBRA REVISION (2)

POWERS & LOGS

(1) Work these out without using a calculator.

(i) $16^{\frac{5}{4}}$ (ii) $\frac{27^{\frac{1}{3}}}{8^{\frac{2}{3}}}$ (iii) $64^{-\frac{2}{3}}$

(2) Solve for x

(i) $9^{x^2+2} = 27^{x+3}$

(ii) $2^{3x-1} = \frac{\sqrt{2}}{8}$

(iii) $2^{2x} - 5(2^x) + 4 = 0$

(iv) $3^{2x+1} - 10(3^x) + 3 = 0$

} Hint: let $y = 2^x$

(3) Work out without using a calculator

(i) $\log_3 9 + \log_4 16$

(ii) $\log_8 16$

(4) Solve for x

(i) $\log_x 125 = 3$

(ii) $\log_x 8 = 3$

(5) Write as a power equation

(i) $\log_{10} 1000 = 3$

(ii) $\log_5 125 = 3$

⑥ Write as a log equation

(i) $3^4 = 81$

(ii) $5^2 = 25$

(iii) $49 = 7^2$

⑦ Write as a single log

(i) $3\log_2 5 + 2\log_2 10 - 4\log_2 8$

(ii) $5\log_3 9 - 2\log_3 27 + \log_3 81$

⑧ Solve for x

(i) $\log_2 x = 3 - \log_2 (x-2)$

(ii) $\log_3 x - \log_3 4 = \log_3 2$

(iii) $\log_2 x = \log_4 (x+6)$

(iv) $e^x = 2$

(v) $\ln x = \frac{1}{2}$

⑨ John has deposited €1000 in an account that pays 6% interest, compounded continuously. The amount, A, after t years is found by $A = P(e^{rt})$, where P is the principal (money at start) and r is the rate written as a decimal. How long will it take for his money to double?

5 MATHS ALGEBRA REV (2) SOLNS

(1) (i) $16^{\frac{5}{4}} = (16^{\frac{1}{4}})^5 = 2^5 = \boxed{32}$

(ii) $\dfrac{27^{\frac{1}{3}}}{(8^{\frac{1}{3}})^2} = \dfrac{3}{2^2} = \boxed{\dfrac{3}{4}}$

(iii) $\dfrac{1}{64^{\frac{2}{3}}} = \dfrac{1}{(64^{\frac{1}{3}})^2} = \dfrac{1}{4^2} = \boxed{\dfrac{1}{16}}$

(2) (i) $(3^2)^{x^2+2} = (3^3)^{x+3}$

$\therefore 3^{2x^2+4} = 3^{3x+9}$

$\therefore 2x^2 + 4 = 3x + 9$

$\therefore 2x^2 - 3x - 5 = 0$

$\therefore (2x - 5)(x + 1) = 0$

$\boxed{x = \frac{5}{2}}$ or $\boxed{x = -1}$

(ii) $2^{3x-1} = \dfrac{2^{\frac{1}{2}}}{2^3}$

$\therefore 2^{3x-1} = 2^{-\frac{5}{2}}$

$\therefore 3x - 1 = -\frac{5}{2}$

$\therefore 3x = 1 - \frac{5}{2} = -\frac{3}{2}$

$\therefore \boxed{x = -\frac{1}{2}}$

(iii) $2^x = y$

$\therefore y^2 - 5y + 4 = 0$

$\therefore (y-4)(y-1) = 0$

$\therefore \boxed{y = 4}$ or $\boxed{y = 1}$

(iv) $3^x = y$

$\therefore (3^x)^2 . 3^1 - 10(3^x) + 3 = 0$

$\therefore 3y^2 - 10y + 3 = 0$

$\therefore (3y - 1)(y - 3) = 0$

$\boxed{y = \frac{1}{3}}$ or $\boxed{y = 3}$

(3) (i) $\log_3 9 + \log_4 16 = 2 + 2 = \boxed{4}$

(ii) $\log_8 16 = x$

$\therefore 16 = 8^x$

$\therefore 2^4 = (2^3)^x \Rightarrow 4 = 3x \quad \therefore \boxed{x = \frac{4}{3}}$

(4) (i) $x^3 = 125$

$\boxed{x = 5}$

(ii) $x^3 = 8$

$\boxed{x = 2}$

(5) (i) $1000 = 10^3$ (ii) $125 = 5^3$

(6) (i) $4 = \log_3 81$ (ii) $2 = \log_5 25$

(iii) $2 = \log_7 49$

(7) (i) $\log_2 5^3 + \log_2 10^2 - \log_2 8^4$

$= \log_2 \frac{(5^3)(10^2)}{8^4}$

(ii) $\log_3 9^5 - \log_3 27^2 + \log_3 81$

$= \log_3 \frac{(9^5)(81)}{27^2}$

(8) (i) $\log_2 x + \log_2 (x-2) = 3$

$\therefore \log_2 x(x-2) = 3$

$\therefore x^2 - 2x = 2^3 = 8$

$\therefore x^2 - 2x - 8 = 0$

$\therefore (x-4)(x+2) = 0$

$\boxed{x = 4}$ or ~~$x = -2$~~

(ii) $\log_3 \frac{x}{4} = \log_3 2$

$\therefore \frac{x}{4} = 2$

$\therefore \boxed{x = 8}$

(iii) $\frac{\log_4 x}{\log_4 2} = \log_4 (x+6)$

$\therefore \frac{\log_4 x}{\frac{1}{2}} = \log_4 (x+6)$

$\therefore 2 \log_4 x = \log_4 (x+6)$

$\therefore \log_4 x^2 = \log_4 (x+6)$

$\therefore x^2 = x + 6$

$\therefore x^2 - x - 6 = 0$

$\therefore (x-3)(x+2) = 0$

$\boxed{x = 3}$ or ~~$x = -2$~~

(iv) $e^x = 2 \Rightarrow x = \log_e 2 = \ln 2 = \boxed{0.693}$

(v) $\ln x = \frac{1}{2} \Rightarrow x = e^{\frac{1}{2}} = \boxed{1.649}$

(9) $2000 = 1000 e^{0.06t}$

$\therefore 2 = e^{0.06t}$

$\therefore \ln 2 = 0.06t$

$\frac{\ln 2}{0.06} = \boxed{t = 11.55 \text{ years}}$