CHRISTMAS REVISION (B) ALGEBRA

① Factorise

(i) $125 - 8p^3$ (ii) $px + qx - p - q$

② Simplify fully

(i) $\frac{x-2}{x-3} + \frac{1}{3-x}$

(ii) $\frac{x-2}{x^2+2x} + \frac{3}{x^2+3x} - \frac{x+4}{x^2+5x+6}$

③ Express r in terms of p and q

(i) $q = \frac{p^2 - rq}{p+r}$ (ii) $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$

④ If $3(x^2+2x)+7 = p(x^2+2) + qx(x-3) + r$ for all values of x, find the values of p, q and r.

⑤ Simplify fully.

$$\frac{1 + \frac{2}{x} - \frac{3}{x^2}}{1 + \frac{3}{x} - \frac{4}{x^2}}$$

⑥ Simplify fully

$$\frac{x^2+8x+15}{x^2-9} \div \frac{xy+5y}{x^2-3x}$$

CHRISTMAS REVISION (B) SOLUTIONS
ALGEBRA

(1) (i) $5^3 - (2p)^3 = \boxed{(5-2p)(25+10p+4p^2)}$

(ii) $px + 2x - p - 2 = x(p+2) - 1(p+2)$

$= \boxed{(p+2)(x-1)}$

(2) (i) $\frac{x-2}{x-3} + \frac{1}{3-x}$

$= \frac{x-2}{x-3} - \frac{1}{x-3}$

$= \frac{x-2-1}{x-3} = \frac{x-3}{x-3} = \boxed{1}$

(ii) $\frac{x-2}{x^2+2x} + \frac{3}{x^2+3x} - \frac{x+4}{x^2+5x+6}$

$= \frac{x-2}{x(x+2)} + \frac{3}{x(x+3)} - \frac{x+4}{(x+3)(x+2)}$

$= \frac{(x-2)(x+3) + 3(x+2) - (x+4)(x)}{x(x+2)(x+3)}$

$= \frac{x^2 - 2x + 3x - 6 + 3x + 6 - x^2 - 4x}{x(x+2)(x+3)}$

$= \boxed{0}$

(3) (i) $2 = \frac{p^2 - r2}{p+r} \Rightarrow 2(p+r) = p^2 - r2$

$\Rightarrow 2p + 2r = p^2 - r2$

$$\Rightarrow 2r + 2r = p^2 - 2p$$

$$\Rightarrow 2qr = p^2 - 2p$$

$$\Rightarrow \boxed{r = \frac{p^2 - 2p}{2q}}$$

(ii) $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$ $\quad \times pqr$

$$\therefore pqr \cdot \frac{1}{p} + pqr \cdot \frac{1}{q} = pqr \cdot \frac{1}{r}$$

$$\therefore qr + pr = pq$$

$$\therefore r(q+p) = pq$$

$$\therefore \boxed{r = \frac{pq}{p+q}}$$

(4) $3(x^2+2x)+7 = p(x^2+2) + qx(x-3) + r$

$$\therefore 3x^2 + 6x + 7 = px^2 + 2p + qx^2 - 3qx + r$$

x^2 coeff

$3 = p + q$

$\therefore 3 = p - 2$

$\boxed{5 = p}$

x coeff

$6 = -3q$

$\therefore \boxed{q = -2}$

constant

$7 = 2p + r$

$\therefore 7 = 2(5) + r$

$\therefore \boxed{-3 = r}$

(5) $$\frac{1+\frac{2}{x}-\frac{3}{x^2}}{1+\frac{3}{x}-\frac{4}{x^2}} \cdot \frac{x^2}{x^2}$$

$$= \frac{x^2+2x-3}{x^2+3x-4} = \frac{(x+3)(x-1)}{(x+4)(x-1)} = \boxed{\frac{x+3}{x+4}}$$

(6) $$\frac{x^2+8x+15}{x^2-9} \div \frac{xy+5y}{x^2-3x}$$

$$= \frac{(x+5)(x+3)}{(x-3)(x+3)} \times \frac{x(x-3)}{y(x+5)}$$

$$= \boxed{\frac{x}{y}}$$