6th YEAR CIRCLE TEST OCT 2019

1. S is the circle

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

(i) Write down the centre and radius of S

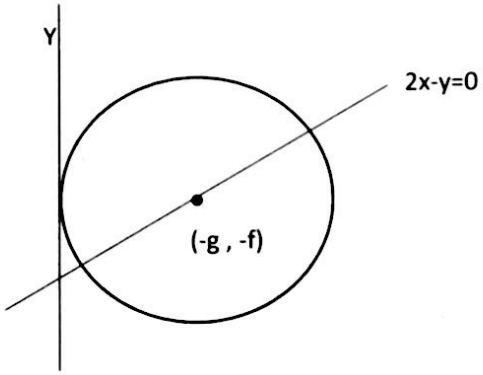
(ii) Write down the equation of a line of slope -1.

(iii) Draw a diagram of the circle S.

(iv) Find the distance from the centre of S to a chord of length $2\sqrt{23}$

(v) Find the equations of the chords of the circle S, of slope -1 and length $2\sqrt{23}$.

2. Find the equations of the circles through the point (2, 14), touching the Y-axis and with their centre on the line $2x - y = 0$.

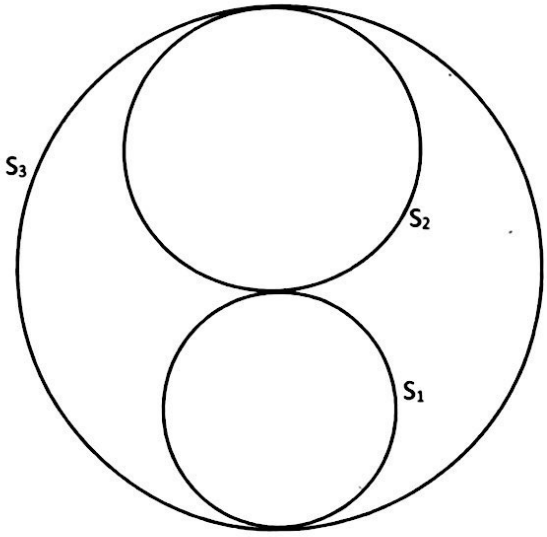


3. Circle S_1 has centre (-1, 3) and radius 3. Circle S_2 has radius 4 and touches S_1 externally at (-1, 6).

(i) Find the centre of S_2.

Circle S_3 is drawn around S_1 and S_2 so that they each touch S_3 internally as shown.

(ii) Find the equation of S_3.



6th YEAR CIRCLE SOLUTIONS

(1) (i) Centre = $\boxed{(2,-3)}$ (5)

$r = \sqrt{4+9+12}$

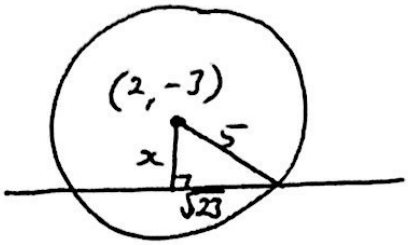
$\boxed{r = 5}$ (5)

(ii) $y = mx + c$

$\therefore y = -x + c$

$\therefore \boxed{x + y + k = 0}$ (5)

(iii)



(iv) $x = \sqrt{5^2 - 23} = \boxed{\sqrt{2}}$ (5)

(v) $\dfrac{|2 \cdot -3 + k|}{\sqrt{1^2 + 1^2}} = \sqrt{2}$ (5)

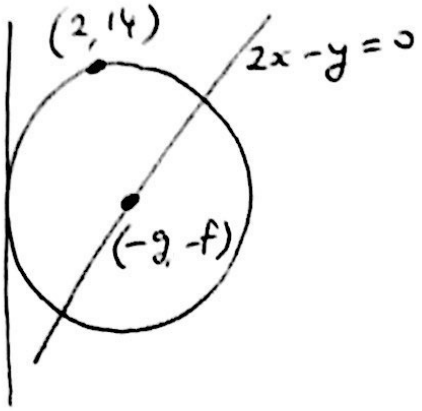
$\therefore \dfrac{|k-1|}{\sqrt{2}} = \sqrt{2}$

$\therefore |k-1| = 2$

$\therefore k - 1 = \pm 2$ $\quad \therefore k = -1$ or $k = 3$ (5)

$\therefore \boxed{x + y - 1 = 0}$ or $\boxed{x + y + 3 = 0}$ (5)

(2)



$x^2 + y^2 + 2gx + 2fy + c = 0$

$(2,14) \Rightarrow \quad 2^2 + 14^2 + 2g(2) + 2f(14) + c = 0$

$\therefore \boxed{200 + 4g + 28f + c = 0} \quad (1)$ (5)

$(-g, -f)$ on line $\Rightarrow \quad 2(-g) - (-f) = 0$

$\therefore \boxed{2g - f = 0} \quad (2)$ (5)

Touches y axis $\Rightarrow \boxed{c = f^2} \quad (3)$ (5)

Sub (2) & (3) in (1)

$\therefore \quad 200 + 2f + 28f + f^2 = 0$

$\therefore \quad f^2 + 30f + 200 = 0$

$\therefore \quad (f + 10)(f + 20) = 0$

$f = -10 \quad$ or $\quad f = -20$

$\therefore g = -5 \qquad g = -10$

$\therefore c = 100 \qquad c = 400$

$\therefore \boxed{\begin{array}{l} x^2 + y^2 - 10x - 20y + 100 = 0 \\ \text{or} \\ x^2 + y^2 - 20x - 40y + 400 = 0 \end{array}}$ (5)

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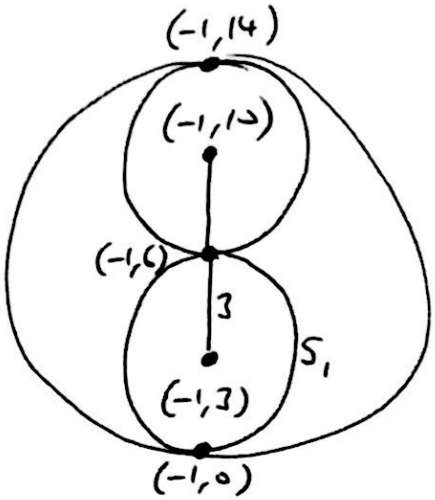

(i) Centre = $\boxed{(-1, 10)}$ (5)

(ii) Radius $S_3 = 7$ (5)

Centre $S_3 = (-1, 7)$ (5)

$\therefore (x+1)^2 + (y-7)^2 = 7^2$

$\therefore \boxed{(x+1)^2 + (y-7)^2 = 49}$ (5)