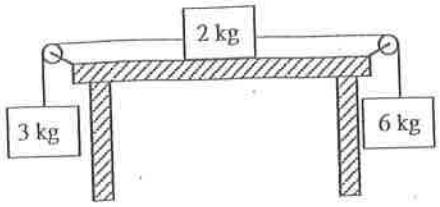
Connected Particles

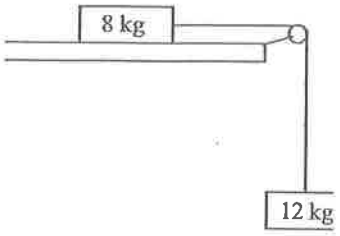
(1) 10) A mass m rests on a rough horizontal table with coefficient of friction μ. It is attached to a second particle of mass $2m$ hanging freely, by means of a light inextensible string passing over a smooth light pulley at the edge of the table. Show that the acceleration of the system is: $\frac{(2-\mu)g}{3}$.

(2) 4) The diagram to the right shows a horizontal table on which a 2 kg mass is placed. It is joined to two other masses, 3 kg and 6 kg respectively, hanging freely on either side of the table, by means of light inextensible strings. The strings pass over smooth light pulleys at the edges of the table. The coefficient of friction on the table is ½. The system released from rest.



(i) Find the common acceleration of the three masses.
(ii) Find their speed after 2 s.

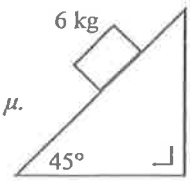
(3) **4.** **(a)** Two particles, of masses 8 kg and 12 kg, are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a smooth horizontal table.



The 12 kg mass hangs freely under gravity.
The particles are released from rest.
The 12 kg mass moves vertically downwards.

(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the acceleration of the 12 kg mass.
(iii) Find the tension in the string.

(4) **(b)** A particle of mass 6 kg is placed on a rough plane inclined at an angle of 45° to the horizontal. The coefficient of friction between the particle and the plane is μ. The particle is released from rest and takes 4 seconds to move a distance of $10\sqrt{2}$ metres down the plane.



(i) Show on a diagram all the forces acting on the particle.

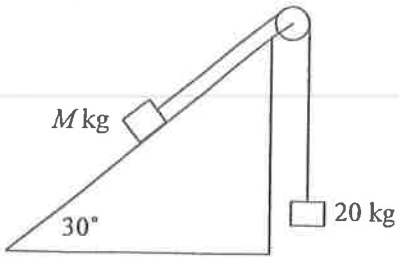
(ii) Show that the acceleration of the particle is $\frac{5\sqrt{2}}{4}$ m/s^2.

(ii) Find the value of μ.

(5) A car of mass 850 kg is climbing a hill inclined at 15° to the horizontal. The engine of the car provides a tractive force of 4500 N. The coefficient of friction between the car and the hill is ¼.

(i) Find the acceleration of the car.
(ii) If the engine of the car is switched off when the car is travelling at 30 m s^{-1}, find how long the car takes to roll to a stop.

4. A particle of mass M kg is placed on a rough plane inclined at 30° to the horizontal.
This particle is connected by a light inextensible string passing over a smooth light pulley at the top of the plane to a particle of mass 20 kg, hanging freely under gravity.
The coefficient of friction between the M kg mass and the plane is $\frac{2}{5\sqrt{3}}$.



The system is released from rest.
The 20 kg mass moves vertically upwards a distance of 16 m in 4 s.

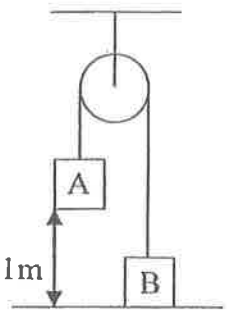
(i) Show on separate diagrams all the forces acting on each particle.

(ii) Show that the constant acceleration of the particles is 2 m/s^2.

(iii) Find the tension in the string.

(iv) Find the value of M.

4. **(a)** A light inextensible string passes over a small fixed smooth pulley.
A particle A of mass 10 kg is attached to one end of the string and a particle B of mass 5 kg is attached to the other end.

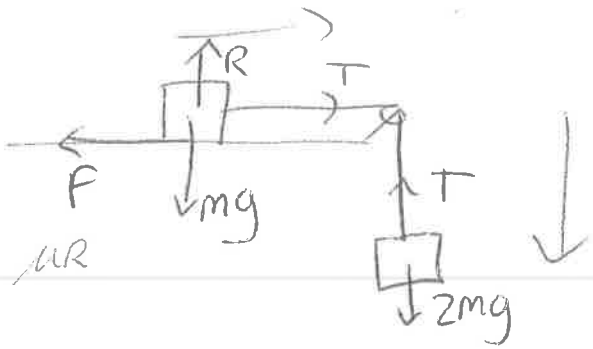


The system is released from rest when B touches the ground and A is 1 m above the ground.

Find **(i)** the speed of A as it hits the ground.

(ii) the height that B rises above the horizontal ground.

2. (1)



$R = mg$

$F = \mu mg$

$T - f = ma$ $\qquad$ $2mg - T = 2am$

$T - \mu mg = ma$ $\qquad$ $2mg - T = 2ma$

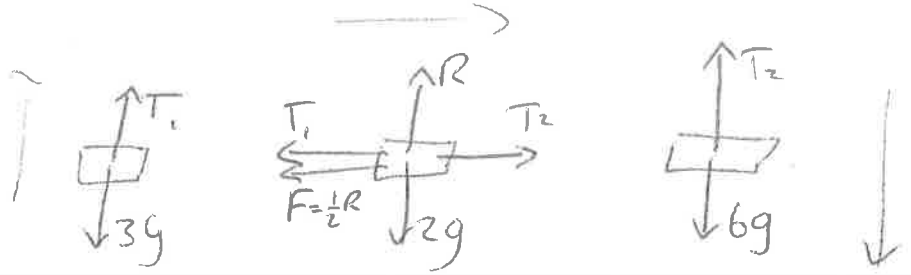
$$2mg - \mu mg = 3ma$$

$$2g - \mu g = 3a$$

$$a = \frac{2g - \mu g}{3}$$

$$= \frac{g(2-\mu)}{3}$$

1.



$R = 2g$

$F = \frac{1}{2}R$

$T_1 - 3g = 3a$ $\quad T_2 - T_1 - g = 2a$ $\quad 6g - T_2 = 6a$

i)

$2g = 11a$

$$a = \frac{2g}{11}$$

ii) $u = 0$, $v = ?$, $t = 2$, $a = \frac{2g}{11}$

$v = u + at$

$= 0 + \frac{2g}{11}(2)$

$$v = \frac{4g}{11}$$

iii) after 2 seconds:

3kg : $u = \frac{4g}{11}$, $a = -g$, $v = 0$, $s =$

$v^2 = u^2 + 2as$

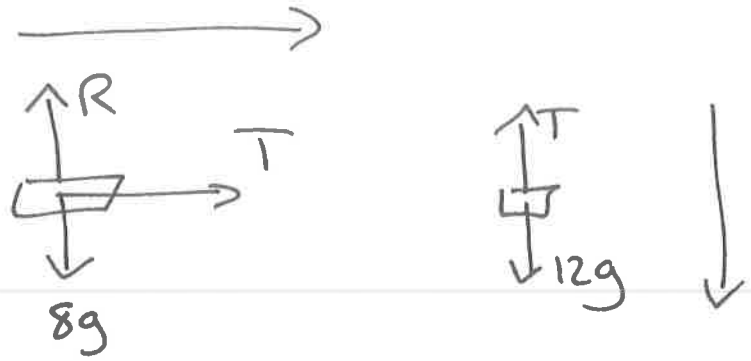
$0 = \left(\frac{4g}{11}\right)^2 - 2(g)(s)$

$2gs = \frac{16g^2}{121}$

$$s = \frac{8g}{121}$$

*

(3)



2004

(5)

$T = 8a$ (1) $\qquad 12g - T = 12a$ (2) (LS)

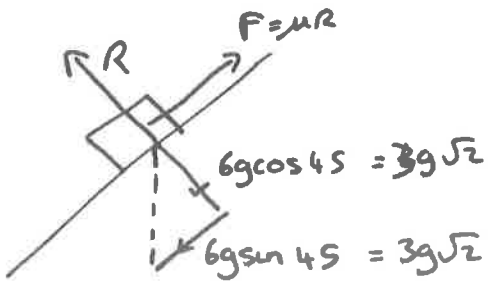
(1) + (2) $\quad = \quad 12g = 20a$

$a = \frac{12g}{20} = \frac{147}{25} = 5.88$ $\quad \backslash\ 32/5$

$T = 8a$

$= 8(5.88)$

$= 47.04$ $\quad$ or $\frac{24g}{5}$ $\quad$ (10)



(5)

$u = 0$
$t = 4$
$s = 10\sqrt{2}$
$a = ?$

$s = ut + \frac{1}{2}at^2$

$10\sqrt{2} = 0 + \frac{1}{2}(a)(16)$

$10\sqrt{2} = 8a$

$\frac{10\sqrt{2}}{8} = a$

$a = \frac{5\sqrt{2}}{4}$ QED

(5)

$\mu = ?$ $\qquad R = 3g\sqrt{2}$ (5)

$3g\sqrt{2} - F = 6\left(\frac{5\sqrt{2}}{4}\right)$

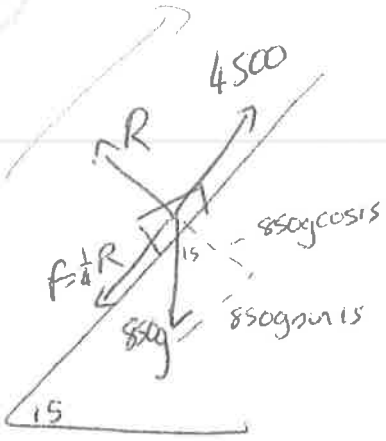
$3g\sqrt{2} - \mu 3g\sqrt{2} = \frac{15\sqrt{2}}{2}$ (5)

$6g\sqrt{2} - 6g\sqrt{2}\mu = 15\sqrt{2}$

$6g - 15 = 6g\mu$

$\mu = \frac{73}{98} = .74$

3. (5)



$\leftrightarrow \quad R = 850g\cos 15 = 8046.2$

$\nearrow \quad 4500 - 850g\sin 15 - \frac{1}{4}(8046.2) = 850a$

$4500 - 2011.5 - 2156.0 = 850a$

$\boxed{a = 0.391}$

ii) $u = 30$

$v = 0$

no 4500 in diagram:

$\swarrow \quad -850g\sin 15 - \frac{1}{4}(8046.2) = 850a$

$-2011.5 - 2156.0 = 850a$

$a = -4.9$

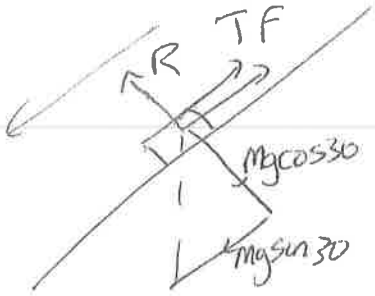
$v = u + at$

$0 = 30 - 4.9t$

$\boxed{t = 6.1}$

6

2005



$s = ut + \frac{1}{2}at^2$

$16 = 0 + \frac{1}{2}a(4^2)$

$2 = a$

$T - 20g = 20a$

$T = 20a + 20g$

$\boxed{T = 236}$

$mg\sin 30 - T - F = Ma$

$\frac{mg}{2} - 236 - \mu mg\cos 30 = m(2)$

$\frac{mg}{2} - 236 - \mu mg\frac{\sqrt{3}}{2} = m2$

$mg - 472 - \mu mg\sqrt{3} = 4m$ $\qquad \mu = \frac{2}{5\sqrt{3}}$

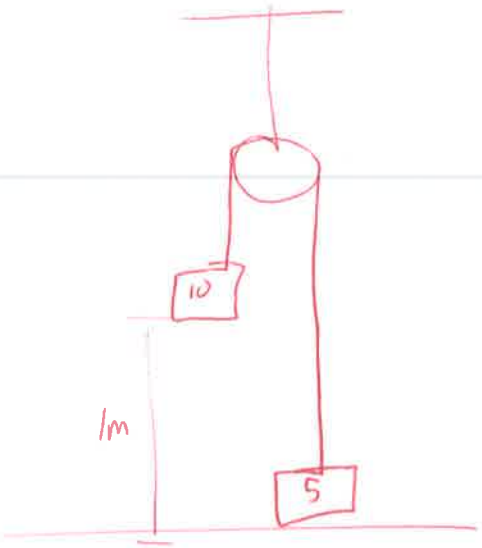
$mg - 472 - \frac{2}{5\sqrt{3}}mg\sqrt{3} = 4m$

$5mg - 2360 - 2mg = 20m$

$m(3g - 20) = 2360$

$m = 251.06$

(7)



$10g - T = 10a$ $\qquad$ $T - 5g = 5a$

Add $\quad 5g = 15a$ $\qquad$ $\boxed{a = \frac{g}{3}}$ 3.27

$T = 5a + 5g = \frac{5g}{3} + \frac{5g}{1} = \frac{5g + 15g}{3} = \boxed{\frac{20g}{3}}$ 2.55

A

$u = 0$
$s = 1$
$a = g/3$
$v = ?$

$v^2 = u^2 + 2as$
$= 0 + 2\left(\frac{g}{3}\right)(1)$
$v^2 = \frac{2g}{3}$

$\boxed{v = \sqrt{\frac{2g}{3}}}$

B

$u = \sqrt{\frac{2g}{3}}$
$v = 0$
$a = -g$
$s = ?$

$v^2 = u^2 + 2as$
$0 = \frac{2g}{3} + 2gs$
$\boxed{s = 1/3}$