

Diff ③ solns

①  $y = x^2 \ln(3x)$

$$\begin{aligned}\frac{dy}{dx} &= x^2 \left( \frac{1}{3x} \cdot 3 \right) + \ln(3x) \cdot 2x \\ &= x + 2x \ln 3x \\ &= \boxed{x(1 + 2 \ln 3x)}\end{aligned}$$

②  $y = e^{2x}$

$$\frac{dy}{dx} = e^{2x} \cdot 2 = 2e^{2x}$$

$$\frac{d^2y}{dx^2} = 2e^{2x} \cdot 2 = 4e^{2x}$$

$$\begin{aligned}\therefore \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y &= 0 \quad ? \\ &= 4e^{2x} - 3(2e^{2x}) + 2(e^{2x}) \\ &= 4e^{2x} - 6e^{2x} + 2e^{2x} \\ &= 0 \quad \checkmark\end{aligned}$$

③  $y = \sin^{-1} 2x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-4x^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{2}{\sqrt{1-0}} = \boxed{2}$$

$$(4) f(x) = (\cos 2x)^3$$

$$\frac{dy}{dx} = 3 (\cos 2x)^2 (-\sin 2x)(2)$$

$$= -6 (\sin 2x) (\cos 2x)^2$$

$$\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{6}} = -6 \sin 2\left(\frac{\pi}{6}\right) (\cos 2\left(\frac{\pi}{6}\right))^2$$

$$= -6 \sin \frac{\pi}{3} (\cos \frac{\pi}{3})^2$$

$$= -6 \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right)$$

$$= \boxed{-\frac{3\sqrt{3}}{2}}$$

$$(5) y = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2} \left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$$

$$\therefore x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - \frac{1}{4}y$$

$$= x^2 \left(-\frac{1}{4} x^{-\frac{3}{2}}\right) + x \left(\frac{1}{2} x^{-\frac{1}{2}}\right) - \frac{1}{4} (x^{\frac{1}{2}})$$

$$= -\frac{1}{4} x^{\frac{1}{2}} + \frac{1}{2} x^{\frac{1}{2}} - \frac{1}{4} x^{\frac{1}{2}}$$

$$= 0$$

