THE FACTOR THEOREM

$(x - k)$ is a factor of a polynomial f (x), iff p $(k) = 0$

Exercise 1.

For each of the factors noted below, write the corresponding root of the polynomial i.e.

Factor = $(x - 2)$, root: $x = 2$

Factor = $(3x + 4)$, root: $x = ?$

(1) $(x + 3)$ (2) $(x - 5)$ (3) $(x + 7)$

(4) $(x - 1)$ (5) $(2x + 1)$ (6) $(3x - 1)$

(7) $(2x - 3)$ (8) $(9x + 4)$ (9) $(3x + 6)$

(10) $(x - 7)$

Exercise 2.

For each of the roots listed below, write the corresponding factor of the polynomial i.e.

$x = 4$ factor = $(x - 4)$

$x = 2/3$ factor = $(3x - 2)$

(1) $x = 5$ (2) $x = 3$ $x = -4$

(3) $x = -2$ (4) $x = ½$ $x = 2/3$

(5) $x = -3/4$ (6) $x = -4/5$

Exercise 3.

Verify that the first expression is a factor of the second expression: -

(1)	$x-2$	$2x^3-5x^2+5x-6$
(2)	$x+3$	$x^3-2x^2-16x-3$
(3)	$x-1$	$2x^3-3x^2-2x+3$
(4)	$x+2$	$3x^3+5x^2-3x-2$
(5)	$2x+1$	$6x^3-x^2-4x-1$

Exercise 4.

Find $\boldsymbol{k}$ if

(1) $x^3+x^2+\boldsymbol{k}x+8$ is divisible by $x-1$

(2) $x^3+\boldsymbol{k}x^2-2x+1$ is divisible by $x+2$

(3) $2x^3-3x^2-\boldsymbol{k}x+7$ is divisible by $2x-1$

(4) $2x^3-5x^2+3x-\boldsymbol{k}$ is divisible by $x-2$

Exercise 5.

Find $\boldsymbol{a}$ if

(1) $(x-1)$ is a factor of $x^3+2x^2-3\boldsymbol{a}x+4$

(2) $(x+2)$ is a factor of $\boldsymbol{a}x^2-2x-3$

(3) $(2x-1)$ is a factor of $4x^2-6x-9\boldsymbol{a}$

(4) $(x+1)$ is a factor of $2x^3-4x^2+3\boldsymbol{a}x-7$

Exercise 6.

Find a and b if

(1) $x^3 + ax^2 - 9x + b$ is divisible by $(x - 1)$ and $(x + 3)$

(2) $(x + 1)$ and $(x - 2)$ are factors of $x^3 + ax^2 + 2x + b$

(3) $(x + 5)$ and $(x - 3)$ are factors of $ax^3 + bx^2 - 58x - 15$

Exercise 7.

(1) If $(x - 1)$ is a factor of $x^3 + 2x^2 - x - 2$, find other *two* factors

(2) If $(x + 3)$ is a factor of $x^3 + 9x^2 + 23x + 15$, find other *two* factors

(3) If $(2x - 1)$ is a factor of $6x^3 + 7x^2 - 9x + 2$, find other *two* factors

(4) If $(x + 3)$ is a factor of $x^3 - 2x^2 + kx + 12$, find k, and other *two* factors

(5) If $(2x - 1)$ is a factor of $2x^3 - 5x^2 - kx +$, find k and *two* other factors

(6) If $(2x + 1)$ is a factor of $2x^3 + kx^2 - 3x - 2$, find k and other *two* factors

Exercise 8.

Much Harder.....

A. If $x^2 - ax - 3$ is a factor of $x^3 - 5x^2 + bx + 9$, find the value of a and b

B. $(x - a)^2$ is a factor of $x^3 + 3px + q$, show that
   (i) $p = a^2$
   (ii) $q = 2a^3$

C. If $(2x - 1)^2 + ax + 15 = 4(x + b)^2$, for all x, find all the possible values of a and b.

D. If $x^2 - p$ is a factor of $ax^3 + 5x^2 + bx + c$, show that $ac = 5b$

E. If $(x - 1)^2$ is a factor of $ax^3 + bx^2 + 1$, find the value of a and b.

SIMULTANEOUS EQUATIONS:

LINEAR: 2 EQUATIONS WITH 2 VARIABLES

Exercise 1. **Solve the Equations: -**

(1) $2x + y = 7$, $2x - y = 3$

(2) $3x + y = 7$, $2x - y = 3$

(3) $2x + y = 8$, $5x - y = 6$

(4) $2x + 3y = 5$, $3x + 4y = 7$

(5) $5x + 4y = 22$, $3x + 5y = 21$

(6) $5x + 3y = 29$, $4x + 7y = 37$

(7) $2x - 5y = 8$, $2x - 3y = 7$

(8) $3x - 4x = -10$, $y + 4x = 8$

(9) $2x - 5y = 3$, $X - 3y = 1$

(10) $3x - 5y = 44$, $5x + 7y = 12$

(11) $3x - 3y = 24$, $^{5x}/_{3} - {}^{y}/_{2} = 12$

(12) $^{x}/_{2} + {}^{y}/_{5} = 4$, $^{x}/_{4} + {}^{y}/_{2} = 6$

(13) $^{x}/_{2} - y = 1$, $x + {}^{y}/_{3} = 9$

(14) $x + y = 1$, $x + 5y/6 = 5/2$

(15) $3x - 6y = 6$, $4y - 5x = -1$

(16) $3x = y - 4$, $3y = 34 - 2x$

(17) $3x - 4 = 5y$, $y - 2x = 2$

(18) $4x + 11 = 3y$, $3(x - 2) - 7y = 0$

(19) $5x - 2y = 7 = 9x - 5y$

(20) $5x + 3y + 1 = 2x + 7y + 3 = 14$

(21) $x + 3y - 3 = 4x - 2y - 5 = 3x - y - 1$

(22) $^{1}/_{x} + {}^{1}/_{y} = 3$, $^{1}/_{x} - {}^{1}/_{y} = 1$

(23) $^{1}/_{x} + {}^{2}/_{y} = 8$, $^{2}/_{x} - {}^{3}/_{y} = -5$

(24) $^{4}/_{x} + {}^{3}/_{y} = -5$, $^{3}/_{x} - {}^{2}/_{y} = 19/4$

(25) *Find 2 numbers whose differences is 11 and one-fifth of whose sum is 9.*

(26) *Four times Brian's age (in years) is 20 greater than Anne's age (in years). One third of Anne's ages is 2 less than Brian's age. Find their ages.*

(27) *Five apples and three oranges cost €5.40. Three applies and five oranges cost €5.80. Find the cost of an apple and the cost of an orange.*

(28) *A year ago, a father was four times as old as his son. Three years ago, he was five times as old as his son. Find the age of the father and the son now.*

(29) *It cost Mr and Mrs Murphy and their two children €64 to go to a show. It cost Mr O'Brien and his four children €68 to go to the same show. Find the cost of entry for one adult and one child.*

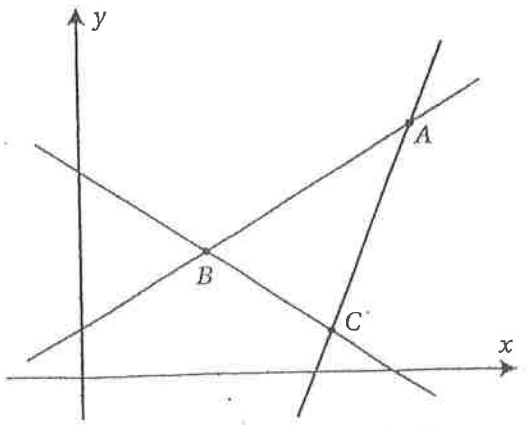
30. Three functions are given by

$f: x \to \frac{3}{5}x + \frac{6}{5}$

$g: x \to \frac{5}{2}x + k$

$h: x \to -\frac{2}{3}x + 5.$

The diagram opposite shows the lines $y = f(x)$, $y = g(x)$ and $y = h(x)$.



(i) Copy the diagram opposite and name each line.

(ii) If the co-ordinates of A are (8,6), find the value of the constant k.

(iii) Find the co-ordinates of the points B and C.

LINEAR: 3 EQUATIONS WITH 3 VARIABLES

Exercise 2. Solve the Equations: -

(1) $x + 2y - z = 1$
$2x - y + 3z = 12$
$3x + 4y + 2z = 19$

(2) $2x - y + 3z = 20$
$3x + 2y - 2z = 1$
$x + y + z = 9$

(3) $3x + y + 2z = -9$
$x - 4y + 5z = -3$
$2x + 3y + 7z = 2$

(4) $2x + 3y - z = 11$
$3x + 4y + 2z = 5$
$x - y - z = 2$

(5) $2x - y - 5z = 27$
$3x + 4y + 2z = -3$
$x + 3y = z = -4$

(6) $x + 2y + 3z = -9$
$2x - 5y - 7z = -1$
$x + 3y - 2z = 2$

(7) $x - 2y + 2z = 13$
$2x + 3y - 5z = -1$
$2x - 4y + 5z = 27$

(8) $x + 3y - 4z = 2$
$3x - 2y + 3z = -13$
$2x + y = -7$

(9) $3x + y + 3z = 0$
$x - y - 2z = -3$
$4x + 2y + 3z = -1$

(10) $2x - 3y + 4z = 0$
$3x - 5y - 5z = 11$
$4x + 7y + 7z = 1$

(11) $x/5 - y/5 + z/2 = -2$
$2x/3 + y/z + z = 5/6$
$x + y - z/2 = 10$

(12) $x + y/z - z/3 = 19/6$
$2x - y + z/2 = 6$
$x/6 - y/2 + z/2 = 0$

(13) $^{x}/_{4} + {}^{y}/_{3} - z = 2$
$^{x}/_{3} - y + {}^{z}/_{3} = 1$
$3x - 2y + 4z = 8$

(14) $^{x}/_{3} - {}^{y}/_{2} + {}^{z}/_{2} = {}^{-1}/_{2}$
$^{x}/_{4} + {}^{2y}/_{3} - {}^{z}/_{3} = {}^{-3}/_{4}$
$^{x}/_{5} - y + {}^{2z}/_{3} = {}^{-4}/_{15}$

(15) $x - 2y = 3$
$2y - z = 1$
$2z + x = -9$

(16) $x = y = -9$
$3x - y + z = -4$
$x - y = 3$

(17) $2x + y - z = 12$
$3x - 2z = 19$
$x + 5z = -5$

(18) $x - 2y = -3$
$2y + z = 3$
$z - 2x = -3$

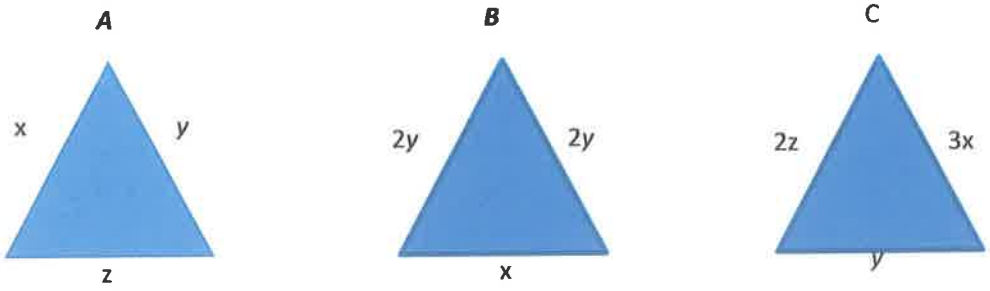
(19) $2x + 3y = x + y + z = 3x + 2y - 4z - 9 = 0$

(20) $x + y = -z,\ y - z = -5,\ x + 27 = 1$

(21) At a food stall in a charity fundraiser, Aimee bought 3 pies, 1 pizza and 1 tart for €22. Ben paid €17 for 1 pie, 1 pizza and 2 tarts, while Chris paid €25 for 2 pies, 2 pizzas and 1 tart. Find the cost of a pie, a tart and a pizza.

(22) A fathers' age (in year) is three times the sum of the ages of his son and daughter. The sum of the father's age and the son's age is nine times the daughter's age. The sum of three ages is 40. Find the age of each person.

(23) The perimeter of triangles ***A***, ***B*** and ***C*** are 12, 19 and 23 cm respectively. Solve for x, y, and z.

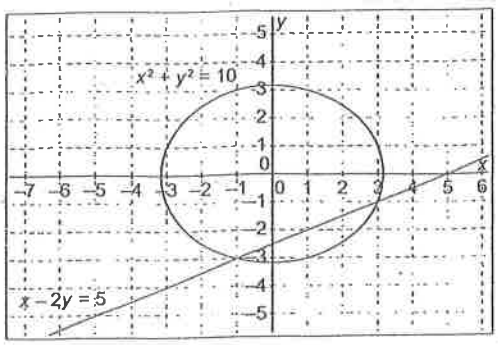


LINEAR AND NON-LINEAR

Exercise 3.

(1) Estimate the points of intersection between the linear and nonlinear graphs shown. Interpret what these points of intersection mean in each case.

(i)



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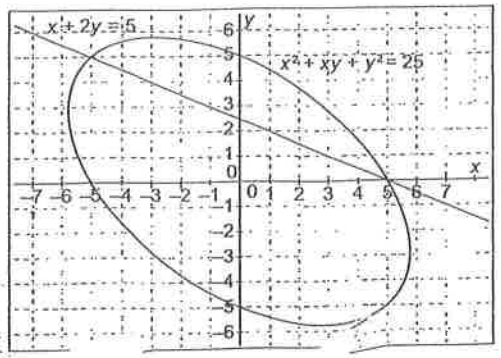

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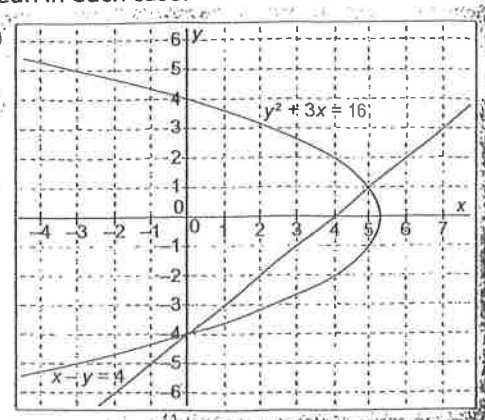

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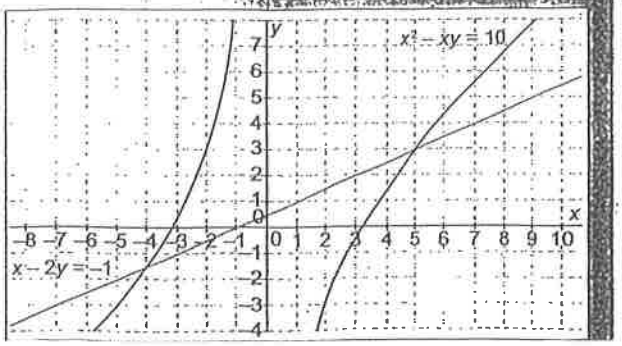

Solve the Equations: -

(2) $x + y = 7$
$xy = 12$

(3) $x - y = 1$
$xy = 42$

(4) $x + y = 3$
$x^2 + y^2 = 5$

(5) $x - y - 1 = 0$
$x^2 + y^2 = 13$

(6) $x^2 + xy + y2 = 52$
$x + y = 8$

(7) $2x - y = 5$
$xy = 0$

(8) $x + y - 6 = 0$
$x^2 + 2y^2 - 24 = 0$

(9) $X + y = 3$
$2/x + 4/y = 4$

(10) $x - 2y = 4$
$x^2 + y^2 = 5$

(11) $3x + y = 10$
$2x^2 + y^2 = 19$

(12) $2x + y + 1 = 0$
$4x^2 + y^2 = 25$

(13) $2x + 3y = -1$
$x^2 + y2 = 25$

(14) f: $x \rightarrow x^2$ and g: $x \rightarrow x + 6$ are *two* functions defined for $x \in \mathbf{R}$.

(i) Sketch the *two* functions on the same diagram, for $-6 < x < 6$

(ii) Use the sketch to determine the values of x for which $f(x) = g(x)$

(iii) Check your answer using simultaneous equations.

(15) A plan is travelling along the line $x + y = 7$. Investigate if it will pass through an ash cloud, given by the equation $x^2 + y^2 = 25$.

(16) A spacecraft is travelling along the line $x - 3y = 0$. Ahead lies a large asteroid field, given by the equation $2x^2 - y = 17$. It is not safe to travel through such an asteroid field. Should the space craft alter its course? Justify your answer.

(17) Show that a plan travelling along the path $x - y + 4 = 0$ will not pass through/touch a cloud given by the equation $x^2 + y^2 = 6$