4th Year

Maths

Booklet 1

Factorising

① Factorising

Ⓐ Highest Common Factor

1. $4x + 8$

2. $22a + 33b$

3. $2ax + 4ay$

4. $6ab - 12bc$

5. $x^2 + x$

6. $5x^2 - 10x$

7. $15x^3 + 10x^2$

8. $6a^2 - 12a$

9. $10x^2 + 40x$

10. $x^3 + x^2 + x$

11. $3x^2 - 6x^2y$

12. $15x^3 - 35x^2$

13. $4ab^2 - 12ab^3$

14. $3x^2 - 9x$

15. $15x^2 - 25x$

Ⓑ Grouping

1. $ax + ay + bx + by$

2. $ac + bc + 3a + 3b$

3. $2ab + 2bc + 3ad + 3cd$

4. $ac - bc + 2a - 2b$

5. $7ax - 7bx + 3a - 3b$

6. $4x - 4y + ax - ay$

7. $x^2 + ax + bx + ab$

8. $4ay + xy - 4az - xz$

9. $2x - 2y - cx + cy$

10. $3x + 3y - bx - by$

11. $2x^2 - 5xy + 4kx - 10ky$

12. $a^2b^2 - b^3 + bc - a^2c$

Ⓒ Quadratics

1. $x^2 + 8x + 7$

2. $x^2 + 8x + 12$

3. $x^2 + 6x + 8$

4. $x^2 + 10x + 16$

5. $x^2 + 13x + 22$

6. $x^2 - 5x + 6$

7. $x^2 - 6x + 8$

8. $x^2 - 8x + 15$

9. $x^2 - 11x + 24$

10. $x^2 - 9x + 18$

11. $x^2 - 2x - 3$

12. $x^2 - x - 12$

13. $x^2 - 9x - 10$

14. $x^2 + 3x - 18$

15. $x^2 + 6x - 40$

(D) DOTS (Difference of two squares)

1. $x^2 - y^2$
2. $x^2 - b$
3. $x^2 - 4^2$
4. $x^2 - 36$
5. $x^2 - 100$

6. $x^2 - 9$
7. $x^2 - 1$
8. $36 - x^2$
9. $100 - y^2$
10. $9x^2 - y^2$

11. $9x^2 - 36$
12. $y^2 - 100$
13. $100x^2 - 1$
14. $4x^2 - 9$
15. $25x^2 - 100$

Leaving Cert :

(E) Mixture of Highest Common Factor and DOTS

1. $3a^2 - 12b^2$
2. $5x^2 - 5y^2$
3. $3a^2 - 27b^2$
4. $3x^2 - 75y^2$

5. $12a^2 - 27$
6. $2x^2 - 32$
7. $20x^2 - 5y^2$
8. $24x^2 - 54$

(F) Harder DOTS

1. $x^2 - (a+b)^2$
2. $9 - (a+b)^2$
3. $25 - (x+5)^2$
4. $16x^2 - (a+p)^2$

5. $(2x+y)^2 - 25$
6. $(5a+c)^2 - 100$
7. $(2x+1)^2 - 9$
8. $(3a+2)^2 - 16$

(G) Harder Quadratics

1. $2x^2 + 5x + 3$
2. $3x^2 + 8x - 3$
3. $2x^2 - 7x + 6$
4. $3x^2 - 4x - 7$
5. $2x^2 - 9x - 5$
6. $5x^2 + 9x - 2$
7. $2x^2 + 7x - 15$

8. $3x^2 - 11x - 20$
9. $7x^2 + 5x - 2$
10. $x^2 - 7x + 12$
11. $x^2 - 2x - 15$
12. $x^2 - 10x + 21$
13. $3x^2 + 11x + 6$
14. $2x^2 + 11x + 14$

15. $4x^2 + 8x + 3$
16. $4x^2 - 11x - 3$
17. $6x^2 - x - 2$
18. $6x^2 - 13x + 2$
19. $8x^2 + 6x - 5$
20. $10x^2 - x - 3$

Higher Level Factorising

SUM & DIFFERENCE OF TWO CUBES

> **Difference of 2 Cubes:** $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
>
> **Sum of 2 Cubes** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Exercise H:

(1) $x^3 + 1$

(2) $1 - x^3$

(3) $y^3 - 8$

(4) $8 - y^3$

(5) $y^3 + 1$

(6) $x^3 + 27$

(7) $x^3 - 125$

(8) $8x^3 + 27$

(9) $125x^3 - 8$

(10) $64m^3 - 27$

(11) $8a^2 + 125y^2$

(12) $64a^3 + 8x^3$

Exercise I: (mixtures)

(1) $4ab^2 - 12ab^3$

(2) $7x^2 + 9x + 2$

(3) $x^2 - 3x - 18$

(4) $3x^2 - 9x$

(5) $5x(2x+3) + 4(2x+3)$

(6) $x^2 - 121$

(7) $2x^2 + 9x - 5$

(8) $4x^2 - 9$

(9) $3ab - 8c + 2b - 12ac$

(10) $8x^2 - 2x - 15$

(11) $15x^2 - 25x$

(12) $2x^2 + 3x + 1$

Exercise J: (more Grouping)

(1) $x^2 + 5x + 3xy + 15y$

(2) $6x^2 + 15ab - 10bx - 9ax$

(3) $2x^2 - 5xy + 4kx - 10ky$

(4) $a^2b^2 - b^3 + bc - a^2c$

(5) $sa^2 + ac - a - 10ab - 2bc + 2b$

(6) $ax - bx - ay + by + a - b$

Exercise K: (harder mixtures)

(1) $x^2 - 9y^2$

(2) $x^{10} - y^{10}$

(3) $6x^2 + x - 2$

(4) $a^2 - 21a - 100$

(5) $14b^2 + 3b - 2$

(6) $y^4 + 2y^2 + 1$

(7) $-4y^2 - 3y + 1$

(8) $18y^2 - 9y + 1$

Exercise L: (harder mixtures)

(1) $x^2 - 9x$

(2) $4x^2 - 100x$

(3) $a^2 + 3ac - 2ab - 6bc$

(4) $y^3 - 2y^2$

(5) $2x^2 + 10x - 28$

(6) $x^2 - xy - x + y$

(7) $9y^4 + 12y^2x^2 + 4x^4$

Exercise M: *(harder dots)*

(1) $9 - (a - b)^2$ **(2)** $25 - (x - 5)^2$ **(3)** $16x^2 - (a + b)^2$

(4) $(2x + y)^2 - 25$ **(5)** $(5a + c)^2 - 100$ **(6)** $(4a + b)^2 - (a + b)^2$

(7) $(3a - 2b)^2 - (2a - 3b)^2$ **(8)** $(4a - y)^2 - (a - y)^2$

(9) $\dfrac{1}{x^2} - \dfrac{1}{4y^2}$

PERFECT SQUARES

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$

Exercise N: (DOTs & CF)

(1) $x^2 - y^2 + ax - ay$ **(2)** $x^2 - y^2 - 3x - 3y$ **(3)** $4x^2 - 9y^2 + 2x + 3y$

(4) $x^2 - a^2 + 8a - 8x$ **(5)** $16y^2 - 9x^2 - 4y - 3x$

Exercise O: (P.S & DOTS)

(1) $x^2 + 2xy + y^2 - a^2$ **(2)** $x^2 + 2xy + y^2 - 4k^2$ **(3)** $a^2 - 2ab + b^2 - x^2$

(4) $b^2 - 2ab + a^2 - 9y^2$

Exercise P: (P.S & CF)

(1) $x^2 + 2xy + y^2 + 7x + 7y$ **(2)** $9a^2 - 6ay + y^2 - 6a + 2y$ **(3)** $4a^2 + 4ab + b^2 + 6a + 3b$

Fractions

Q Divide the following:

1. $\dfrac{x^{10}}{x^6}$ 2. $\dfrac{15y^{10}}{y^5}$ 3. $\dfrac{14a^4}{7a^3}$ 4. $\dfrac{9z^{10}}{9z}$

5. $\dfrac{-x^{10}}{x^8}$ 6. $\dfrac{-12a^4}{6a^3}$ 7. $\dfrac{9a^7}{-3a^4}$ 8. $\dfrac{-8m^4}{-2m^2}$

9. $\dfrac{-6c^{10}}{-3c^7}$ 10. $\dfrac{x^{12}y^{10}}{x^2y^2}$ 11. $\dfrac{x^{10}a^4}{x^8a^2}$ 12. $\dfrac{y^2p^2}{yp}$

13. $\dfrac{p^2c^4}{p^2c}$ 14. $\dfrac{4x^4y^3}{2xy}$ 15. $\dfrac{9ac^{10}}{3ac^6}$ 16. $\dfrac{-4a^{10}c^4}{2a^9c^2}$

17. $\dfrac{-12x^{10}y^{14}}{6x^2y^2}$ 18. $\dfrac{15a^7x^4}{-3ax^4}$ 19. $\dfrac{-25d^7y^4}{5d^6y}$ 20. $\dfrac{-20a^{10}x^7y^9}{-10a^3x^4y^3}$

R Factorise top line and Divide the following:

1. $\dfrac{6x-9}{3}$ 2. $\dfrac{8x-4}{4}$ 3. $\dfrac{-2x-4}{2}$ 4. $\dfrac{4x-10y}{2}$

5. $\dfrac{-14x-21y}{7}$ 6. $\dfrac{xy+x}{x}$ 7. $\dfrac{a^7+a^5}{3}$ 8. $\dfrac{y^{10}-y^7}{y^4}$

9. $\dfrac{x^2+x}{x}$ 10. $\dfrac{y^3+y^5}{y^3}$ 11. $\dfrac{x^3+x^2+3x}{x}$

12. $\dfrac{9x^2+6x}{3x}$ 13. $\dfrac{4y^2+8y}{4y}$ 14. $\dfrac{15x^2+15x}{5x}$

15. $\dfrac{9y^5+6y^4}{3y^3}$ 16. $\dfrac{27a^3-18a^2}{9a^2}$

(S) <u>Simplify, by factorising top or bottom first:</u>

1. $\dfrac{x^2-1}{x-1}$

2. $\dfrac{x+2}{x^2-4}$

3. $\dfrac{4x^2-64}{3x^2-12x}$

4. $\dfrac{x^3-1}{x-1}$

5. $\dfrac{x^2+2x+4}{x^3-8}$

6. $\dfrac{x^2-1}{x^3-1}$

7. $\dfrac{x^3-8}{2x^2-8}$

8. $\dfrac{3x^3-3}{(x-1)^2}$

9. $\dfrac{15x^2+45x+30}{4x^2+8x+4}$

10. $\dfrac{10x^2-1000}{50x^2+500x}$

11. $\dfrac{4x^2-36}{x^2-6x+9}$

12. $\dfrac{2x^3+16}{x^2-2x+4}$

13. $\dfrac{4x^3-32}{x^3-4x^2+4x}$

14. $\dfrac{5x^2-20x+20}{5x^2-20}$