End of 5th YR Summer Rev

Impacts (1)

Direct

4th

(i) Two smooth spheres, A and B, of equal radii, have masses 4 kg and 8 kg respectively. They lie at rest on a smooth horizontal floor so that the line joining their centres is perpendicular to the vertical wall. A is projected towards B with speed u and collides with B. B then hits the wall, rebounds and collides with A again. This final collision reduces B to rest. If the coefficient of restitution between A and B is $\frac{1}{4}$, calculate

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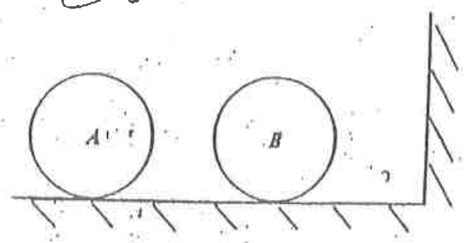

(i) the coefficient of restitution between B and the wall. $\frac{2}{7}$

(ii) the final velocity of A in terms of u. $u/14$

(ii) the total loss of energy due to the three collisions. $\frac{195}{98}u^2$

$e = \frac{1}{7}$

1st

(2) State the laws governing oblique collisions between two smooth elastic spheres.
Two such spheres A and B of mass 5 and 10 kg respectively, collide obliquely. The coefficient of restitution is $\frac{1}{7}$.
Immediately before collision the velocity of A is $5\vec{i} + 4\vec{j}$ and that of B is $-2\vec{i} - 3\vec{j}$, where speeds are in m/s and $\vec{i}$ and $\vec{j}$ are unit vectors along and penpendicular to the line of centres.
Find the velocity of (i) A (ii) B immediately after impact. (i) $-\frac{1}{3}\vec{i} + 4\vec{j}$
Show that the loss of kinetic energy is 80 J. (ii) $\frac{7}{3}\vec{i} - 3\vec{j}$
Calculate the tan of the angle through which B is deflected after the collision. $\tan^{-1}\frac{24}{23}$

2nd

(3) State the laws governing the oblique collision of smooth elastic spheres.
Two smooth elastic spheres A and B of mass 4 kg and 8 kg respectively, collide obliquely. The coefficient of restitution is 0·4. Before collision the velocity of A is $(3\vec{i} + 4\vec{j})$ m/s and that of B is $(-4\frac{1}{2}\vec{i} - p\vec{j})$ m/s where $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the line of centres at the moment of impact.

(i) Find the velocity of each sphere after impact. $\vec{V}_A = -4\vec{i} + 4\vec{j}$

(ii) Show that the loss of kinetic energy, as a result of the impact, is 63 J. $\vec{V}_B = -\vec{i} - p\vec{j}$

(iii) If after impact the spheres are moving at right angles to each other, calculate the value of p.

$p = 1$

$e = \frac{1}{2}$

1st

(4) 8. State the laws governing the oblique collision between smooth elastic spheres.
Two smooth spheres p and q of masses $2k$ and k, respectively, collide obliquely and the coefficient of restitution for the collision is $\frac{1}{2}$. The velocity of p before impact is $2v\vec{i} + 5v\vec{j}$ and the velocity of q before impact is $-4v\vec{i} + 3v\vec{j}$, where $\vec{i}$ points along the line of centres at impact. Find the velocities of the spheres after the impact and show that the loss in kinetic energy is $9\ kv^2$.

$\vec{v}_p = -v\vec{i} + 5v\vec{j}$, $\vec{v}_q = 2v\vec{i} + 3v\vec{j}$

1.

	m	u	v
A	4	u	x
B	8	0	y

$e = \frac{1}{4}$

PCM : $4u + 8(0) = 4x + 8y$

$$\boxed{4u = 4x + 8y}$$

Newton : $\frac{x-y}{u-0} = \frac{-1}{4}$

$$\boxed{4x - 4y = -u}$$

A hits B

$4x + 8y = 4u$
$4x - 4y = -u$

$12y = 5u$

$y = \frac{5u}{12}$, $x = \frac{u}{6}$

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	m	u	v
B	8	$\frac{5u}{12} \rightarrow$	z

e

PCM : one object ✗

B hits Wall

Newton : $\cancel{z = 5u}$

$\frac{z}{\frac{5u}{12}} = -e$

$$\boxed{z = -\frac{5ue}{12}}$$ ie $\leftarrow$

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	m	u	v
A	4	$u/6$	m
B	8	$-\frac{5u}{12}e$	0

B hits A

$e = \frac{1}{4}$ PCM : $4\left(\frac{u}{6}\right) + 8\left(\frac{-5ue}{12}\right) = 4m + 8(0)$

$8u - 40ue = 48m$ ✱

$2u - 10ue = 12m$

$$\boxed{u - 5ue = 6m}$$

Newton : $\dfrac{m-0}{\frac{u}{6}+\frac{5ue}{12}} = -\dfrac{1}{4}$

$$4m = -\left[\frac{u}{6}+\frac{5ue}{12}\right]$$

$$\boxed{48m = -2u - 5ue} \quad *$$

$$48m = 8u - 40ue$$
$$48m = -2u - 5ue$$

$$8u - 40ue = -2u - 5ue$$

$$10\cancel{u} = 35\cancel{u}e$$

$$\frac{10}{35} = e$$

$$\frac{2}{7} = e$$

- Final Velocity of A = ? $\quad m = ?$

$$6m = u - 5ue$$

$$6m = u - 5u \cdot \frac{2}{7}$$

$$6m = u - \frac{10u}{7}$$

$$= \frac{7u - 10u}{7}$$

$$6m = -\frac{3u}{7}$$

$$m = -\frac{\cancel{3}u}{7 \cdot \cancel{6}\,2} = -\frac{u}{14}$$

direction.

KE_{lost}

	m	u	v
A	4	u	$u/14$
B	8	0	0

$$KE_{before} = \tfrac{1}{2}(4)(u^2) + \tfrac{1}{2}(8)(0^2) = 2u^2$$

$$KE_{after} = \tfrac{1}{2}(4)\left(\frac{u^2}{14^2}\right) + \tfrac{1}{2}(8)(0^2) = \frac{2u^2}{196}$$

lost/gain!

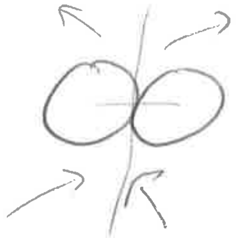
$$\Delta KE = \frac{2u^2}{196} - \frac{2u^2}{1} = \frac{2u^2 - 392u^2}{196} = \frac{-390u^2}{196} = \frac{195u^2}{98}$$

2.

	m	u	v
A	5	$5\vec{i} + 4\vec{j}$	$p\vec{i} + 4\vec{j}$
B	10	$-2\vec{i} - 3\vec{j}$	$q\vec{i} - 3\vec{j}$

$e = \frac{1}{7}$

obliquely



$\vec{j}$ – unchanged

PCM :
$$5(5) + 10(-2) = 5(p) + 10(q)$$
$$25 - 20 = 5p + 10q$$
$$5p + 10q = 5$$
$$\boxed{p + 2q = 1}$$

Newton :
$$\frac{p - q}{5 + 2} = -\frac{1}{7}$$
$$7p - 7q = -7$$
$$\boxed{p - q = -1}$$

$$\begin{array}{r} p + 2q = 1 \\ p - q = -1 \\ \hline 3q = 2 \end{array}$$
$$q = 2/3$$

$$p = -1 + q$$
$$= -\frac{3}{3} + \frac{2}{3} = -\frac{1}{3}$$

final Velocities :
A : $-\frac{1}{3}\vec{i} + 4\vec{j}$

B : $\frac{2}{3}\vec{i} - 3\vec{j}$

$KE_{lost} = ?$

	m	$\lvert u\rvert$	$\lvert v\rvert$
A	5	$\sqrt{25+16}$	$\sqrt{\frac{1}{9}+16}$
B	10	$\sqrt{4+9}$	$\sqrt{\frac{4}{9}+9}$

$KE_{before} = \frac{1}{2}(5)\sqrt{(41)} + \frac{1}{2}(10)(13)$

$= \cancel{127.5}\ 102.5 + 65$

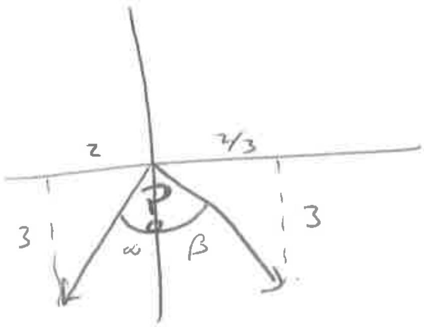
$= \cancel{192.5}\ 167.5$

$KE_{after} = \frac{1}{2}(5)\left(16\,{}^{1}/_{9}\right) + \frac{1}{2}(10)\left(9\,{}^{4}/_{9}\right)$

$= 40\frac{5}{18} + 47\,{}^{2}/_{9}$

$= 87.5$

$KE_{lost} = \cancel{105}\ 80$



$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{2/3 + 2/9}{1-\left(\frac{2}{3}\right)\left(2/9\right)} = \dfrac{18+6}{27-4}$

$= \dfrac{24}{23}$

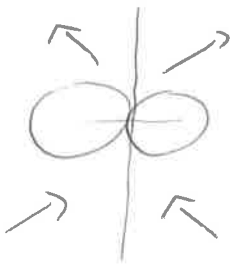
$\tan^{-1}\left(\frac{24}{23}\right)$

3.

	m	u	v
A	4kg	$3\vec{i} + 4\vec{j}$	$q\vec{i} + 4\vec{j}$
B	8kg	$-\frac{9}{2}\vec{i} - p\vec{j}$	$r\vec{i} - p\vec{j}$

$e = 0.4$

oblique



$\vec{j}$ - unchanged

PCM : $4(3) + 8\left(-\frac{9}{2}\right) = 4(q) + 8(r)$

$12 - 36 = 4q + 8r$

$\boxed{4q + 8r = -24}$

Newtons : $\frac{q - r}{3 + \frac{9}{2}} = -0.4$

$\frac{q - r}{7.5} = -0.4$

$\boxed{q - r = -3}$ $\times 4$

$4q + 8r = -24$
$4q - 4r = -12$

$12r = -12$

$\boxed{\begin{matrix} r = -1 \\ q = -4 \end{matrix}}$

final velocities : A : $-4\vec{i} + 4\vec{j}$

B : $-\vec{i} - p\vec{j}$

ii) loss in K.E

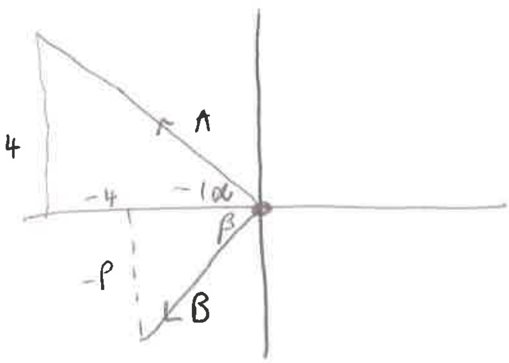
	m	$\lvert u\rvert$	$\lvert v\rvert$
A	4	$\sqrt{3^2+4^2}$	$\sqrt{4^2+4^2}$
B	8	$\sqrt{\left(\frac{9}{2}\right)^2+p^2}$	$\sqrt{1^2+p^2}$

$$KE_{before} = \tfrac{1}{2}(4)(25) + \tfrac{1}{2}(8)\left(\tfrac{81}{4}+p^2\right)$$
$$= 50 + 81 + 4p^2$$
$$= \boxed{131 + 4p^2}$$

$$KE_{after} = \tfrac{1}{2}(4)(32) + \tfrac{1}{2}(8)(1+p^2)$$
$$= 64 + 4 + 4p^2$$
$$= \boxed{68 + 4p^2}$$

$$KE_{lost} = (68+4p^2) - (131+4p^2) = \underline{63\,J} \quad \text{QED}$$

iii) After impact : moving at 90° to each other



$\tan\alpha = \frac{4}{4} = 1$

$\alpha = 45°$

$\tan\beta = \frac{p}{1} = p$ $\quad \beta = \tan^{-1}\left(\frac{p}{1}\right)$

$90° = 45° + \beta$

$\Rightarrow \beta = 45°$

$\tan 45 = \frac{p}{1}$

$1 = \frac{p}{1}$

$p = 1$

better

$-\vec{i} + 4\vec{j}$ $\quad m = \frac{4}{4} = -1$

$-\vec{i} - p\vec{j}$ $\quad m = \frac{-p}{-1} = p$

$90° \Rightarrow \perp \Rightarrow m_1 \times m_2 = -1$

$(-1)(p) = -1$

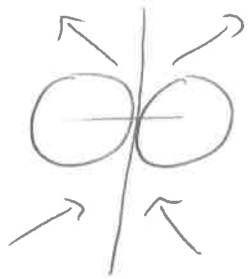
$\boxed{\Rightarrow p = 1}$

4

	m	u	v
p	$2k$	$2v\vec{i} + 5v\vec{j}$	$p\vec{i} + 5v\vec{j}$
q	k	$-4v\vec{i} + 3v\vec{j}$	$q\vec{i} + 3v\vec{j}$

$e = \frac{1}{2}$

obliquely :



J - unchanged

PCM : $2k(2v) + k(-4v) = 2k(p) + k(q)$

$$4\cancel{k}v - 4\cancel{k}v = 2\cancel{k}p + \cancel{k}q$$

$$\boxed{0 = 2p + q}$$

Newton : $\dfrac{p - q}{2v + 4v} = -\dfrac{1}{2}$

$$2p - 2q = -2v - 4v$$

$$\boxed{2p - 2q = -6v}$$

$$2p + q = 0$$
$$2p - 2q = -6v$$
$$3q = 6v$$
$$\boxed{q = 2v}$$

$$2p + 2v = 0$$
$$2p = -2v$$
$$\boxed{p = -v}$$

Final Velocities : p : $-v\vec{i} + 5v\vec{j}$

q : $2v\vec{i} + 3v\vec{j}$

$KE_{lost} = ?$

	m	$\lvert u \rvert$	$\lvert v \rvert$
P	$2k$	$\sqrt{4v^2+25v^2}$	$\sqrt{v^2+25v^2}$
Q	k	$\sqrt{16v^2+9v^2}$	$\sqrt{4v^2+9v^2}$

$$KE_{before} = \tfrac{1}{2}(2k)(29v^2) + \tfrac{1}{2}(k)(25v^2)$$

$$= 29kv^2 + 12.5kv^2$$

$$= \boxed{41.5kv^2}$$

$$KE_{after} = \tfrac{1}{2}(2k)(26v^2) + \tfrac{1}{2}(k)(13v^2)$$

$$= 26kv^2 + 6.5kv^2$$

$$= \boxed{32.5kv^2}$$

$$KE_{lost} = 9kv^2$$