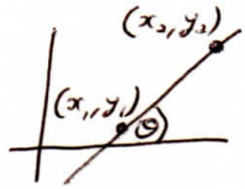
PAPER II

THE LINE

① Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

② Slope = $m = \tan\theta = \dfrac{y_2-y_1}{x_2-x_1}$



③ Collinear points ⇒ on same st. line ⇒ same slope between them.

④ Eqn of line : (a) $y - y_1 = m(x - x_1)$

(b) $ax + by + c = 0$ ⇒ $m = -\dfrac{a}{b}$ (slope)

⑤ // lines ⇒ same slope

⊥ lines ⇒ $m_1 \times m_2 = -1$ (turn slope upside down & change sign to find ⊥ slope)

⑥ Slope of line = 2

⇒ $2x - y + k = 0$

Use given pt to find k

eg $(3, -1)$ on L ⇒ $6 + 1 + k = 0$ ⇒ $k = -7$

∴ $2x - y - 7 = 0$ is eqn of L.

⑦ Line cuts X-axis ⇒ $y = 0$

Line cuts Y-axis ⇒ $x = 0$

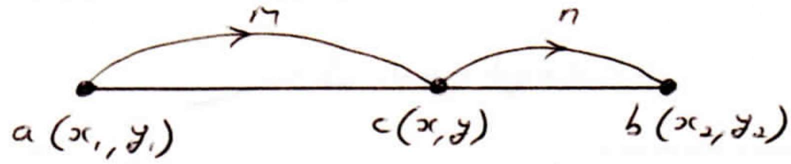
⑧ Intersecting lines ⇒ sim. eqns to find pt. of intersection

⑨ Area of a triangle = $\frac{1}{2}|x_1y_2 - x_2y_1|$

where vertices are $(0,0)$, (x_1, y_1) and (x_2, y_2)

MUST move △ to get one vertex at $(0,0)$

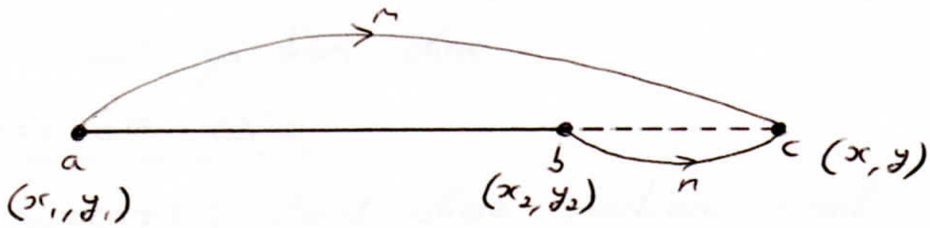
(10) INTERNAL DIVISION OF LINE SEGMENT :



$$\boxed{x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}}$$

$I \Rightarrow +$

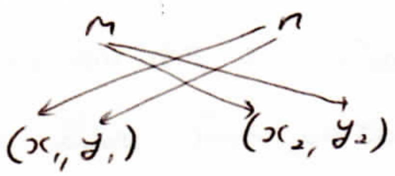
EXTERNAL DIVISION OF LINE SEGMENT :



$$\boxed{x = \frac{mx_2 - nx_1}{m-n}, \quad y = \frac{my_2 - ny_1}{m-n}}$$

$E \Rightarrow -$

ie

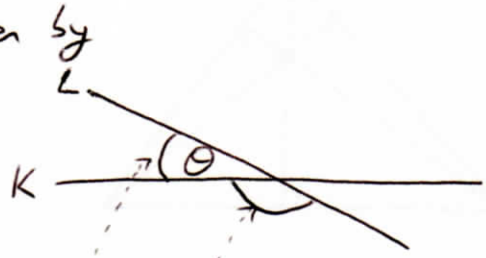


$I \Rightarrow +$

$E \Rightarrow -$

(11) ANGLE BETWEEN 2 LINES given by

$$\boxed{\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}}$$



$\pm$ gives us 2 answers : one acute
one obtuse

(a) To find acute angle say $|\tan\theta| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

(b) To remember formula see p9 for $\tan(A - B)$ formula
NB

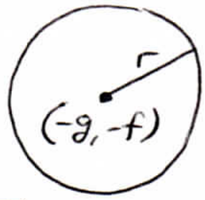
(c) If asked for eqn of line making angle θ with other line, then call slope of new line m, and use formula.

THE CIRCLE

① EQUATIONS

(i) Centre $(-g, -f)$, Radius r $\Rightarrow$ $\boxed{x^2 + y^2 + 2gx + 2fy + c = 0}$

$\therefore \boxed{r = \sqrt{g^2 + f^2 - c}}$



(ii) Centre (h, k), Radius r $\Rightarrow$ $\boxed{(x-h)^2 + (y-k)^2 = r^2}$

MAKE SURE NUMBERS IN FRONT OF x^2 and y^2 ARE BOTH = 1 BEFORE READING OFF CENTRE & RADIUS !!

② POINT INSIDE/OUTSIDE/ON

PIC < 0 $\Rightarrow$ INSIDE

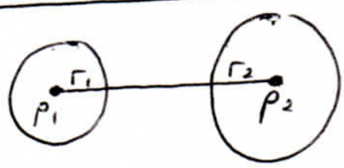
PIC = 0 $\Rightarrow$ ON

PIC > 0 $\Rightarrow$ OUTSIDE

PIC: put point in circle eqn.

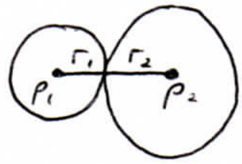
③ INTERSECTING CIRCLES

(i)



$|p_1p_2| > r_1 + r_2$

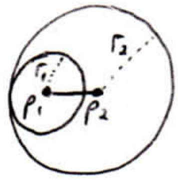
(ii)



external touch

$|p_1p_2| = r_1 + r_2$

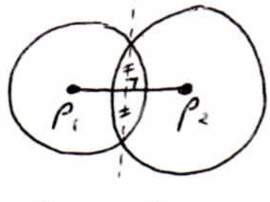
(iii)



internal touch

$|p_1p_2| = r_2 - r_1$

(iv)

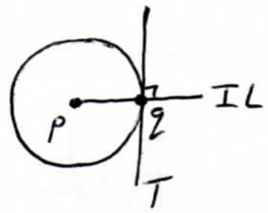


$|p_1p_2| < r_1 + r_2$

Common chord bisected by line of centres.

④ TANGENTS

DON'T FORGET TO DRAW THE IL



IL $\perp$ T

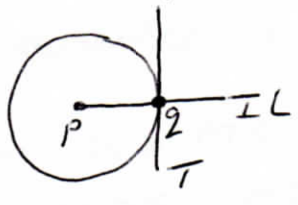
(i) TANGENT AT : Find slope of IL
$\Rightarrow$ Find slope of T
q on T $\Rightarrow$ Find eqn of T

(ii) GIVEN SLOPE OF T : Write down general form of T
eg slope of 2 $\Rightarrow$ $2x - y + k = 0$
PIL centre with T, answer = radius
$\Rightarrow$ find k and sub back in.

(iii) TANGENT FROM : Write down eqn of T as
$mx - y + k = 0$ (ie slope m)
PIL centre with T, answer = radius
$\Rightarrow$ find 2 possible values of m

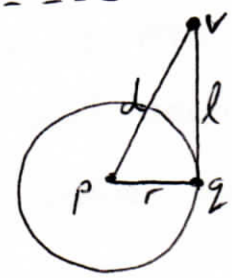
(iv) To show a line is a tangent to a circle :
PIL centre with line = r

(v) To find point of contact of circle and tangent :



Simultaneous eqns of
T and IL.

(vi) LENGTH OF TANGENT FROM A POINT

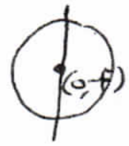


$d^2 = l^2 + r^2$ Pythagoras.
$\Rightarrow l = \sqrt{d^2 - r^2}$

(iv) GIVEN CENTRE ON A LINE :

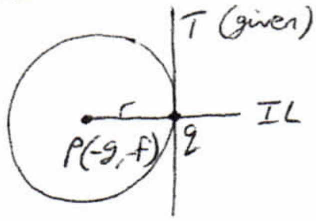
(a) Centre on X-axis $\Rightarrow f=0$

(b) Centre on Y-axis $\Rightarrow g=0$



(c) Centre on random line
$\Rightarrow$ sub $(-g,-f)$ into eqn of line

(v) GIVEN IT TOUCHES A LINE :



Use :
$r = $ PIL p with T

$IL \perp T \Rightarrow m_{IL} = -\dfrac{1}{m_T}$

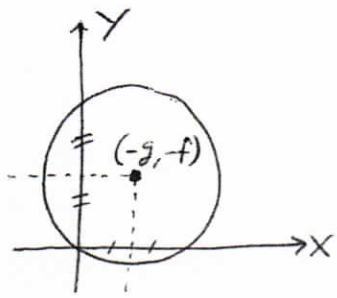
& simultaneous eqns.

NB
- Touches X-axis $\Rightarrow c=g^2$
- Touches Y-axis $\Rightarrow c=f^2$
- Touches both axes $\Rightarrow c=f^2=g^2$

Pictures will help.

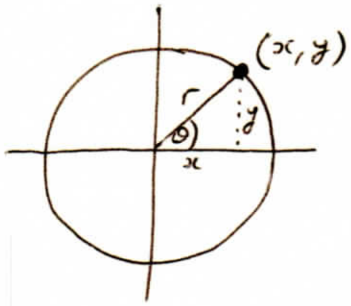
(vi) GIVEN IT INTERCEPTS THE AXES :

Remember IL bisects a chord



TRIG SUMMARY

(1)



$$\cos\theta = \frac{x}{r}$$

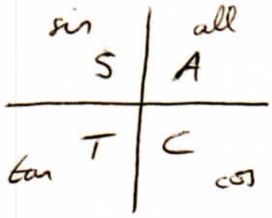
$$\sin\theta = \frac{y}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

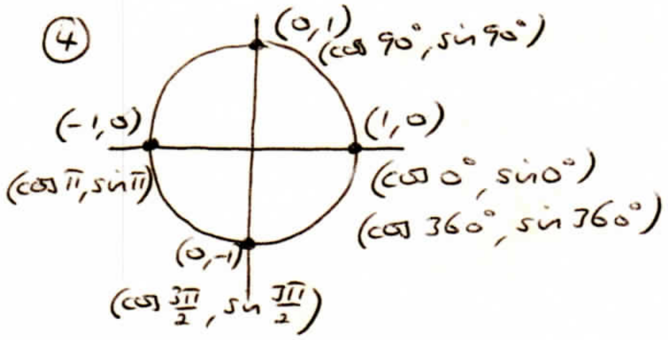
(2) All angles measured ANTICLOCKWISE.

(3)



Tells us where sin, cos or tan are positive.

(4)

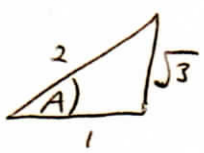


(see p13 tables)

(5) If they give you a trig function as a fraction, use Pythagorus to find the others

eg $\sec A = 2 \Rightarrow \frac{1}{\cos A} = 2 \Rightarrow \cos A = \frac{1}{2}$

If A acute ⇒



$\Rightarrow \tan A = \sqrt{3}$ etc.

Use  for negative / positive results.

⑥ Well behaved angles

$180°$ and $360°$

eg $\cos(180° + \theta) = -\cos\theta$

$\sin(360° + \theta) = \sin\theta$

$\tan(360° - \theta) = -\tan\theta$ etc.

Badly behaved angles

$90°$ and $270°$ → Try not to use these if possible

eg $\cos(90° + \theta) = -\sin\theta$

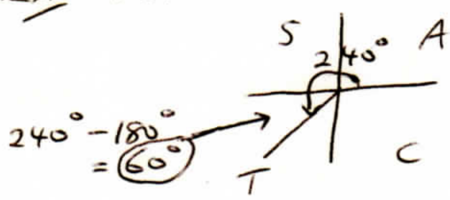
$\sin(270° + \theta) = -\cos\theta$ etc.

⑦ Negative angles :

$$\left.\begin{array}{ll} \cos(-\theta) = \cos\theta & \text{rolls out} \\ \sin(-\theta) = -\sin\theta & \text{filters out} \\ \tan(-\theta) = -\tan\theta & \text{filters out} \end{array}\right\} \text{ see p13 tables}$$

⑧ To find sin/cos/tan of angles in $2^{nd}/3^{rd}/4^{th}$ quad. look up angle relative to $180°$ or $360°$ & use ASTC.

Ex Find $\sin 240°$



$\sin 60° = \frac{\sqrt{3}}{2}$

3^{rd} quad $\Rightarrow$ sin −ve

$\therefore \sin 240° = \boxed{-\frac{\sqrt{3}}{2}}$

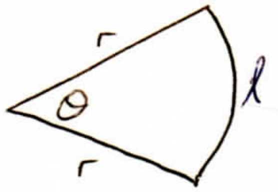
⑨ RADIANS

$\boxed{\pi \text{ Rad} = 180°}$

2π Rad $= 360°$

$\Rightarrow 270° = \frac{270°}{180°} \times \pi = \frac{3\pi}{2}$ etc.

(10)



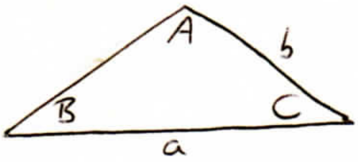
Arc length $= l = r\theta$ [see p9]

θ in rads

Area $= A = \frac{1}{2}r^2\theta$ [see p9]

θ in rads

(11) SINE RULE



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ [p16]

Use this if you know an angle and the opposite side.

NB $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$\Rightarrow \sin A = ak$, $\sin B = bk$, $\sin C = ck$

can be useful in proofs.

(12) COS RULE

Know the proof !!

$a^2 = b^2 + c^2 - 2bc\cos A$ [p16]

or $b^2 = a^2 + c^2 - 2ac\cos B$

or $c^2 = a^2 + b^2 - 2ab\cos C$

- Only use it if you can't use the sin rule.
- Start with the side you want to find or the side opposite the angle you want.

(13) 3-D TRIANGLES

- Separate out all the triangles you can see & draw 2-D pictures for each relevant one.
- Look out for right-angled-triangles.

(14) TRIG IDENTITIES

[See proofs sheet]

(i) Page 13/14 is essential

(ii) Anything on p13/14 can be used in the proof of another identity.

(iii) Main examples:
- $\cos^2 A + \sin^2 A = 1$
- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin 2A = 2 \sin A \cos A$

etc.

(iv) If they give you a half angle eg $\frac{A}{2}$ then let $\frac{A}{2} = X$ to get rid of fraction

$\Rightarrow A = 2X$ etc.

(v) To prove identities:
- Start with LHS & simplify
- Then simplify RHS
- Never move stuff from LHS to RHS... keep the sides separate
- Get everything into sin and cos if necessary eg $\tan A = \dfrac{\sin A}{\cos A}$ etc.
- Let $\frac{A}{2} = X$ if half angles appear.
- Look out for factors if necessary.

(15) CONVERSIONS

(a) PRODSUMS (p15) (useful in integration)

eg $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

etc.

- A and B can be *anything*. Just sub them into the formula.
- Be careful with the "2" in front.

(b) SUMPRODS (p15)

eg $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

etc.

- Again, A and B can be *anything*.

(16) TRIG EQUATIONS

You have to SOLVE these.

STEPS: (i) Try to get same angle everywhere.

(ii) Try to combine into a single trig function.

(iii) Bring everything to one side, 0 on other if necessary.

(iv) Factorise if possible

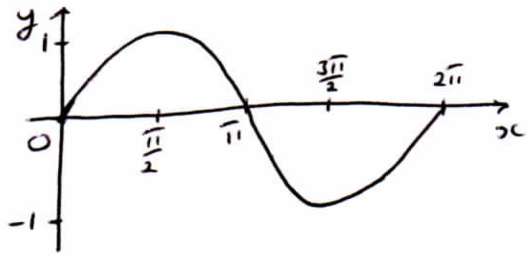
(v) Use sumprod formula if necessary

(vi) Let $\frac{A}{2} = X$ for half-angles

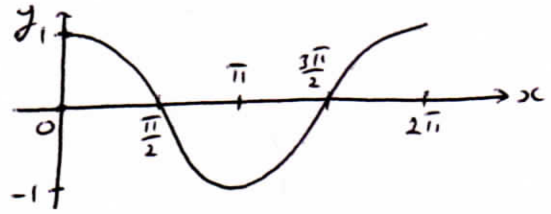
(vii) Make sure to get *all* possible solutions by using ASTC up as far as question allows eg $0 \leq A \leq 360°$.

(17) GRAPHING TRIG FNS

- Be able to plot $y = a\sin n\theta$ and $y = a\cos n\theta$



$y = \sin x$



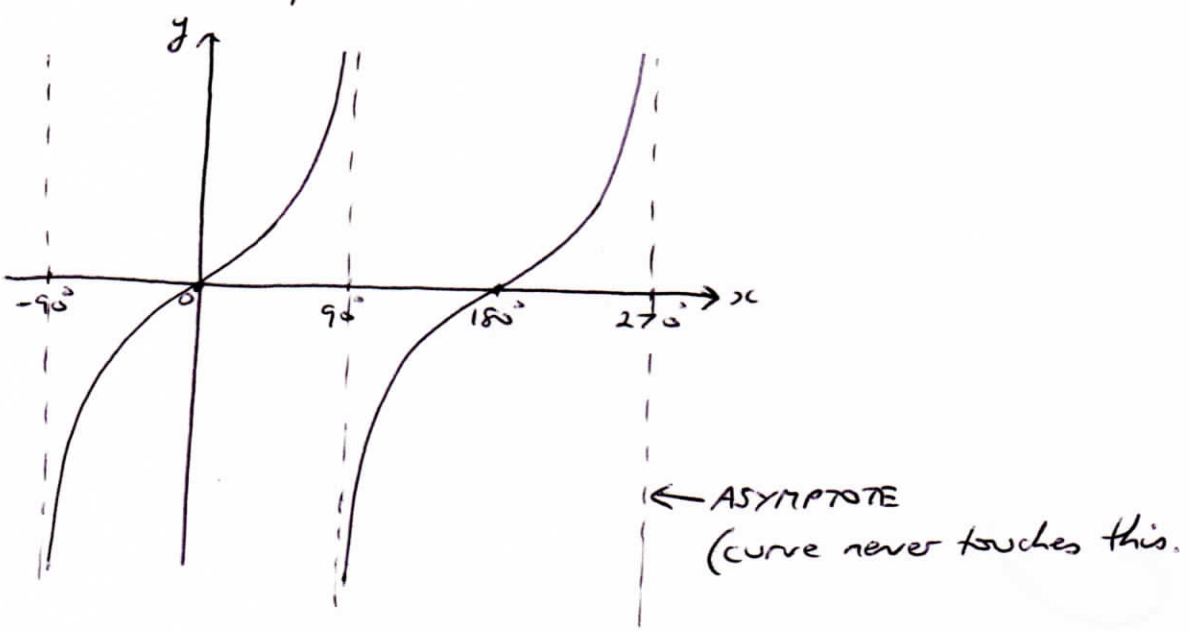
$y = \cos x$

etc.

- Range $= [-1, 1]$ (from lowest to highest on y-axis)
- Period $= 2\pi$ or $360°$ (how often on x-axis does graph repeat)

- $y = a\sin n\theta$ or $y = a\cos n\theta$
  - → If 'a' increases, the range increases
  - → If "n" increases, the period decreases

- Be able to plot $y = \tan x$



PROBABILITY

1. ARRANGEMENTS

(i) Draw boxes → one box for each place to fill

(ii) Fill in restrictions first.

(iii) Multiply the numbers in the boxes.

(iv) Number of arrangements of n things, no repeats $= n!$

(v) Number of arrangements of n different objects taken r at a time, no repeats $= {}^nP_r = \frac{n!}{(n-r)!}$

(vi) Objects must be beside each other ⇒ treat them as one object (stuck together)

(vii) Number of arrangements of n objects, p alike of one kind, q alike of another kind $= \frac{n!}{p!\,q!}$

2. CHOICES

(i) Number of selections of n different objects taking r at a time $= {}^nC_r = \boxed{\binom{n}{r} = \frac{n!}{r!\,(n-r)!}}$

(ii) Again, build in obvious restrictions at start.

(iii) Be careful where order is important
⇒ mixture of arrangements & choices type.

(iv) Quick way for $\binom{n}{r}$

eg $\binom{10}{3} = \frac{10.9.8}{3.2.1}$

no. of numbers on top & bottom.

③ PROBABILITY

(i) $P(E) = \dfrac{\text{Number of desired outcomes}}{\text{Total number of possible outcomes}}$

(ii) $\boxed{P(\text{not } A) = 1 - P(A)}$

Be careful that "A" is the exact opposite of "not A".

(iii) A probability is always between 0 and 1
impossible certainty.

(iv) Relative frequency = Experimental probability

$= \dfrac{\text{Number of successful trials}}{\text{Total number of trials}}$

(v) Expected frequency = Probability × number of trials.

(vi) MUTUALLY EXCLUSIVE ⇒ cannot happen together
(at same time)

(vii) EITHER/OR RULE :

$\boxed{P(\text{Either } A \text{ or } B) = P(A) + P(B) - P(A \cap B)}$

ie or means ADD

(Be careful of overlap between A and B)

(viii) VENN DIAGRAMS :

Always fill in from the middle outwards.

$P(A \cup B) = P(A \text{ or } B)$

$P(A \cap B) = P(A \text{ and } B)$

$(A \cup B)' =$ not in A or B

(ix) AND RULE : (for <u>independent</u> events)

$$\boxed{P(A \text{ and } B) = P(A) \times P(B)}$$

ie <u>AND</u> means <u>MULTIPLY</u>

(x) INDEPENDENT EVENTS mean they have no effect on each other.

INDEPENDENT $\neq$ MUTUALLY EXCLUSIVE !!!

(xi) $P(A|B)$ = Probability of A given B

$$\boxed{P(A|B) = \frac{\#(A \cap B)}{\# B} = \frac{P(A \cap B)}{P(B)}}$$

(xii) AND RULE : (for <u>any</u> events)

$$\boxed{P(A \text{ and } B) = P(A) \times P(B|A)}$$

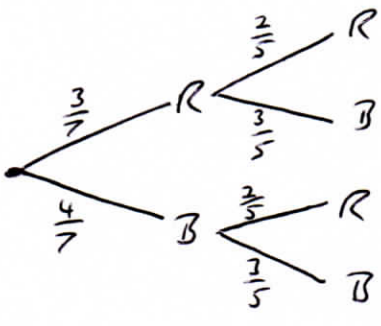
(xiii) To show if A and B are independent,

show <u>one</u> of :

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \cap B) = P(A) \times P(B)$

(xiv) Be able to draw TREE DIAGRAMS

eg



$\Rightarrow RR = \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$

$\Rightarrow RB = \frac{3}{7} \times \frac{3}{5} = \frac{9}{35}$

$\Rightarrow BR = \frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$

$\Rightarrow BB = \frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$

TOTALS ADD TO <u>1</u>.

(xv) EXPECTED VALUE

$$E(x) = \sum x \,.\, P(x)$$

- ie multiply each outcome by the prob. of that outcome and add up your answers.
- Used to see if a game is fair or not
- If expected payout is zero $\Rightarrow$ game is fair

take cost of game into account.

(xvi) BERNOULLI TRIALS :

- $P(r \text{ successes in } n \text{ trials}) = \binom{n}{r} p^r q^{n-r}$

where p = prob. of "success" (p33 tables)

q = prob. of "failure"

(xvii) Be able to use $\binom{n}{r}$ within probability Q's.

(xviii) $P(\text{at least one}) = 1 - P(\text{none})$ etc.

STATISTICS

① TYPES OF DATA

(i) NUMERICAL (numbers) → DISCRETE (can only take certain values)
→ CONTINUOUS (can take any value)

(ii) CATEGORICAL → NOMINAL (no obvious order to categories)
→ ORDINAL (have an order)

(iii) PRIMARY ⇒ collected by person who uses it

(iv) SECONDARY ⇒ collected by someone else

(v) UNIVARIATE vs BIVARIATE
1 variable / 2 variables

② COLLECTING DATA

(i) CENSUS ⇒ everyone in population is used to get data

(ii) SURVEY ⇒ only some people are used to get data

(iii) QUESTIONNAIRES
→ No "leading" questions
→ Make sure "fair" not "biassed"

(iv) CONTROL GROUP ⇒ eg in medical study to compare results with group using the "real" medicine.

(v) DESIGNED EXPERIMENTS on people.
→ EXPLANATORY VARIABLE (controlled) vs RESPONSE VARIABLE (observed)

③ SAMPLING (for a survey)

(i) Make sure to avoid bias in who you select.

(ii) SIMPLE RANDOM SAMPLE ⇒ each member of population has equal chance of being selected.

(iii) STRATIFIED SAMPLE ⇒ population split into different groups. Same proportion of each group selected randomly.

(iv) SYSTEMATIC SAMPLE ⇒ choose people/things at regular intervals (eg every 10th)

(v) QUOTA SAMPLE ⇒ population split into groups. Certain number selected by interviewer from each group.

(vi) CLUSTER SAMPLE ⇒ population split into groups. Some clusters are picked randomly and <u>everyone</u> in those clusters is looked at.

(vii) CONVENIENCE SAMPLE ⇒ pick easiest people to find (eg 1st 100 people)

④ "AVERAGES"

(i) MODE = most common value

(ii) MEAN = $\dfrac{\text{sum of nos.}}{\text{number of nos.}} = \mu$

(iii) MEAN of freq. distribution = $\dfrac{\sum fx}{\sum f}$ (use table of results) (p33 tables)

(iv) MEDIAN = middle number (when they're in <u>order</u>)

(if n numbers, middle one is $\frac{1}{2}(n+1)^{th}$)

(if there's 2 middle numbers, get their average)

⑤ VARIABILITY

ie how spread out from the mean is the data

(i) RANGE = largest value − smallest value

(ii) LOWER QUARTILE = value $\frac{1}{4}$ way into data (in order)

UPPER QUARTILE = value $\frac{3}{4}$ way into data (in order)

INTERQUARTILE RANGE = UPPER QUARTILE − LOWER QUARTILE

(iii) STANDARD DEVIATION (formula p 33 tables)

(a) $\sigma = \sqrt{\dfrac{\sum (x-\mu)^2}{n}}$ or $\sigma = \sqrt{\dfrac{\sum f(x-\mu)^2}{\sum f}}$

for list of numbers — for frequency table

(b) Use calculator → make sure you know how !!

(iv) PERCENTILES

→ divide data into 100 equal parts

→ "I'm on the 60th percentile" means 60% of people got less than me in the test.

→ To work out percentiles : P_k

(a) Order numbers small to big

(b) Then find k % of the number of data values

ie $n \times \frac{k}{100}$

(c) If you get a whole number, eg 8, then P_k is $\frac{8^{th} + 9^{th}}{2}$

If you don't get a whole number, eg $8\frac{1}{4}$, then $P_k = 9^{th}$ value.

⑥ STEM & LEAF PLOTS

eg

0	6 9
1	2 5 7
2	3 3 6 8
3	0 2 7
4	1 2 6
5	3

This means 17.

Make sure leaves are ordered.

$\boxed{\text{Key } 3|2 = 32}$

Back-to-back stemplot

eg

girls		boys
4 3 2	2	1
8 7 6 5	3	2 5 6
5 3	4	7 9 5
1	5	3 6

Key 6|3 means 36.

key 4|7 = 47

Be able to work out median, mode and interquartile range from a stemplot.

⑦ HISTOGRAMS

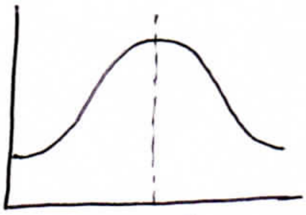
Like bar charts, but area of bar = frequency.

(We only draw bars with equal width)

Bars join together.

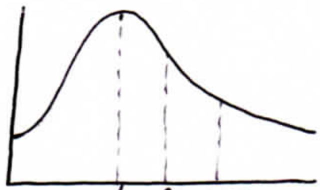
⑧ SHAPE OF DISTRIBUTION

(i) SYMMETRICAL (NORMAL)



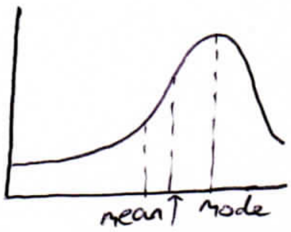
mean
= median
= mode

(ii) POSITIVE SKEW (TAIL TO RIGHT)

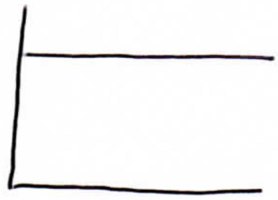


mode ↑ mean
median

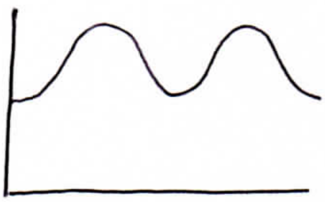
(iii) NEGATIVE SKEW
(TAIL TO LEFT)



(iv) UNIFORM

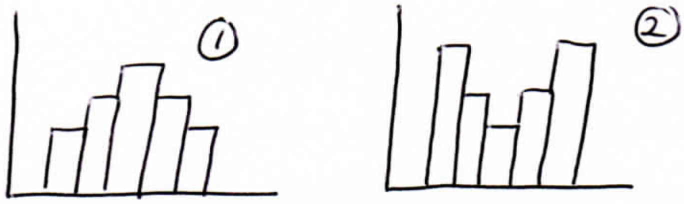


(v) BIMODAL



These pictures can all be drawn using histograms instead of curves

NB



② has bigger σ than ① even though they have same μ.

⑨ THE NORMAL DISTRIBUTION

(a) EMPIRICAL RULE

- 68% of data lies between $\mu - \sigma$ and $\mu + \sigma$
- 95% lies between $\mu - 2\sigma$ and $\mu + 2\sigma$
- 99.7% lies between $\mu - 3\sigma$ and $\mu + 3\sigma$

(b) STANDARD SCORES (Z-SCORES)

- Used to compare results from different data sets
- $z = \dfrac{x - \mu}{\sigma}$ (p34 tables)

z tells you how many standard devs from μ you are

(c) Normal distribution drawn using z-scores is called the STANDARD NORMAL CURVE.

(p36 tables)

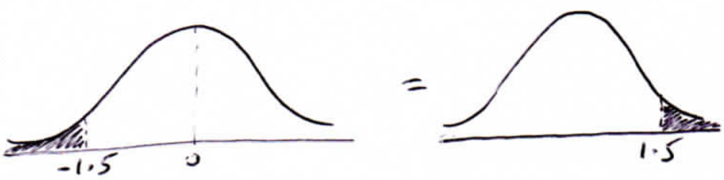
(d) • Area under section of standard normal curve = probability of result being in that section

• Area under whole curve = 1

• Other areas given on p36 tables

• Look out for using symmetry of the curve.

ie



etc.

(10) SCATTER PLOTS

(a) They're just graphs like in science.
Make sure to label the axes

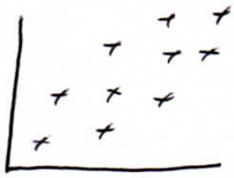
(b)

positive correlation

negative correlation

no correlation

strong positive correlation



weak positive correlation

(c) Sometimes correlation means a "CAUSAL RELATIONSHIP" (ie one thing causes another thing) but not always.

(d) CORRELATION COEFFICIENT (r)

• Use calculator to calculate it

• $r = 0 \Rightarrow$ no correlation

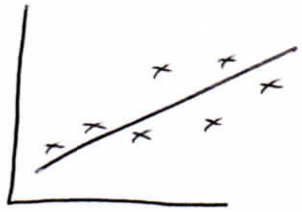
• $r = 1 \Rightarrow$ perfect positive correlation

• $r = -1 \Rightarrow$ perfect negative correlation

(e) LINE OF BEST FIT

- Straight line as close as possible to all points
- Goes through $(\bar{x}, \bar{y})$ (average of x's, ave of y's)
- Doesn't have to go through $(0,0)$

eg.



- To find the equation get slope and a point on the line etc.

(11) INFERENTIAL STATS

STANDARD ERROR

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \qquad p$$

(can use $\hat{p}$ instead)

p = population proportion

$\hat{p}$ = sample proportion

MARGIN OF ERROR (at the 95% level of confidence)

$$= 1.96\,\sigma_{\hat{p}}$$

$$= 1.96\sqrt{\frac{p(1-p)}{n}}$$

$$\approx \frac{1}{\sqrt{n}}$$

CONFIDENCE INTERVAL

$$\hat{p} - 1.96\sigma_{\hat{p}} \le p \le \hat{p} + 1.96\sigma_{\hat{p}}$$

We can say with 95% confidence that the true proportion, p, lies within this interval.

HYPOTHESIS TESTING

Tests whether or not claims being made are likely to be true.

H_0 = null hypothesis = claim being made or statement of no change

H_A = alternative hypothesis = opposite of H_0

H_0 is assumed to be true unless proven not to be true (like in court)

- Work out margin of error
- Write down confidence interval
- If claim lies within this interval it is upheld. If not, then it is rejected.

CENTRAL LIMIT THEOREM

- The distribution of the sample means is normal.

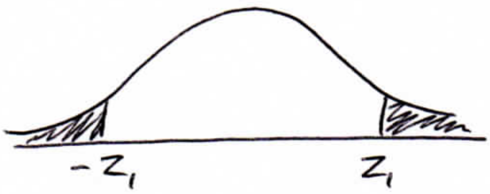
$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ are mean and standard deviation of the sample mean.

- Use $\frac{\sigma}{\sqrt{n}}$ whenever question talks about sample means.

P - VALUES

= sum of the 2 shaded regions



ie p-value $= 2(1 - P(z \leq z_1))$

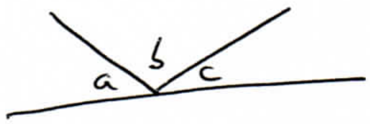
If $p \leq 0.05$, reject H_0.

If $p > 0.05$, do not reject H_0.

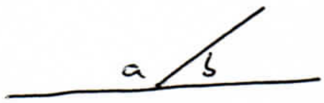
(The 0.05 comes from 5% level of significance, or 95% confidence level again)

GEOMETRY

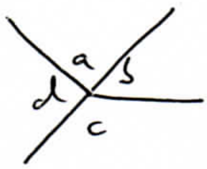
A) Results from JC for triangles and //ograms

(1) 

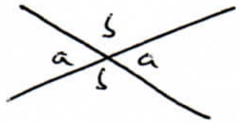
$a+b+c = 180°$

(2) 

$a+b = 180°$ (a & b are SUPPLEMENTARY ANGLES)

(3) 

$a+b+c+d = 360°$

(4) 

VERTICALLY OPPOSITE.

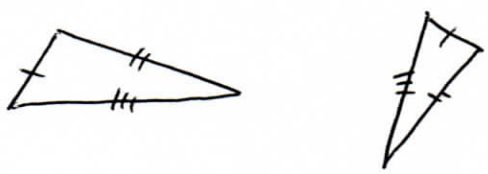
etc.

see book (T & T) p 79 + 80

B) Congruent triangles

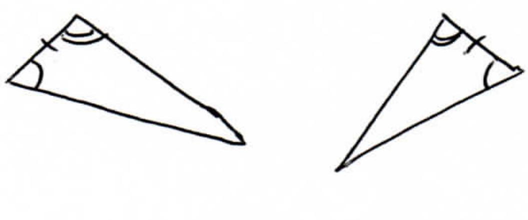
Triangles are CONGRUENT if one of these is true

(i) SSS

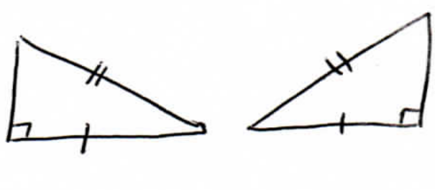


(ii) SAS

(iii) ASA



(iv) RHS



c) Triangle theorems

① **Theorem 7**

Biggest side opposite biggest angle
Smallest side " smallest " .

② **Theorem 8**

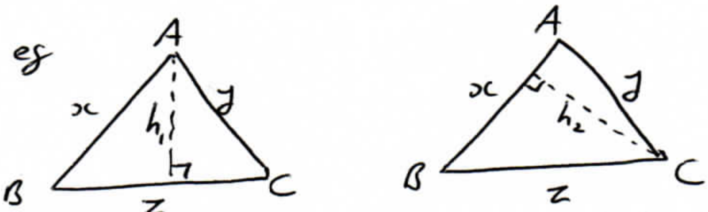
Two sides of a Δ are together greater than 3^{rd} side.

③ **Theorem 16**

For any triangle, base times height does not depend on the choice of base.

(since area same each way

eg



Area $= \frac{1}{2}(z)(h_1)$ Area $= \frac{1}{2}(x)(h_2)$ $\therefore zh_1 = xh_2$ etc)

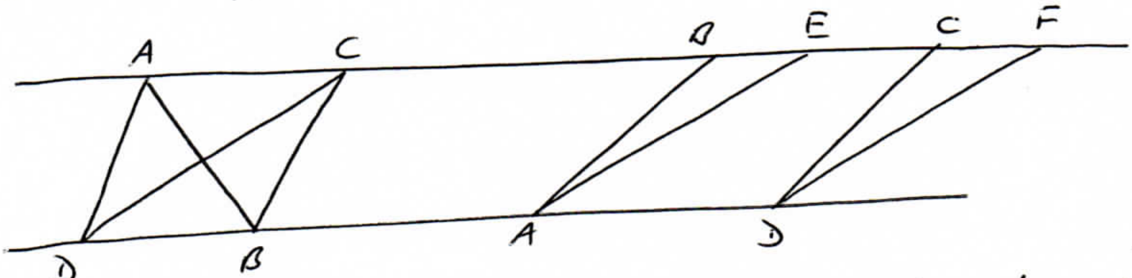
④ **Theorem 17**

Diagonal of a parallelogram bisects the area.

⑤ **Theorem 18**

Area of parallelogram = base × ⊥ height.

⑥ Parallelograms (or triangles) with the same base and between the same // lines have same area

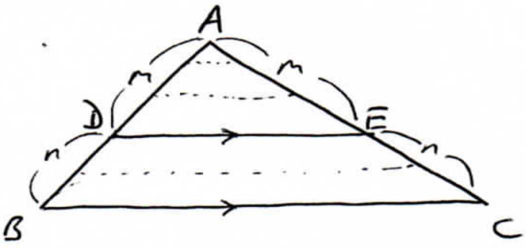


Area ABC = Area ADC Area ABCD = Area AEFD.

D) RATIO THEOREMS

① Theorem 12 [proof required : see later]

A line drawn parallel to one side of a triangle divides the other two sides in the same ratio.

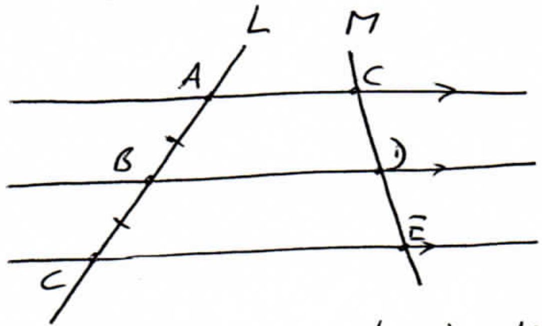


ie $\dfrac{|AD|}{|DB|} = \dfrac{|AE|}{|EC|}$ etc

② Theorem 11 [proof required : see later]

If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

ie



L & M are called transversals

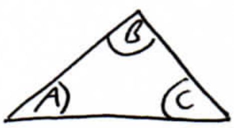
$|AB| = |BC|$ $\quad$ $|CD| = |DE|$

③ Similar triangles

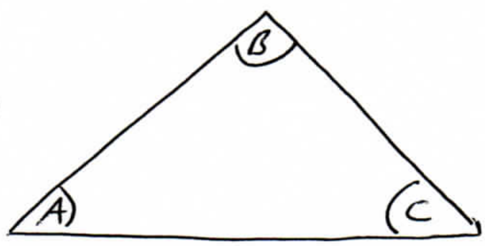
= equiangular triangles

ie same angles in both triangles

eg



is similar to



PTO

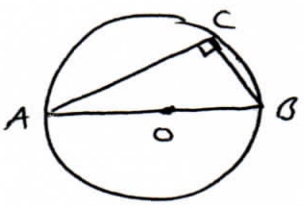
Theorem 13 [proof required : see later]

If two triangles ABC and DEF are similar, then their sides are proportional, in order

$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

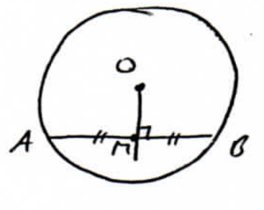
E) CIRCLE THEOREMS

(1)



Angle on diameter = 90°

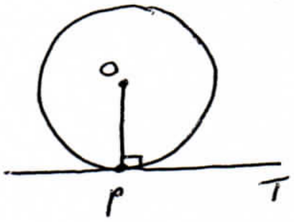
(2)



Theorem 21

The perpendicular from the centre of a circle to a chord bisects the chord.

(3)

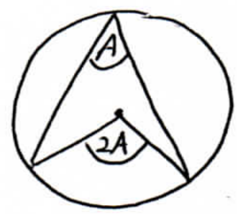


Theorem 20

(i) A tangent is ⊥ to the radius that goes to the point of contact.

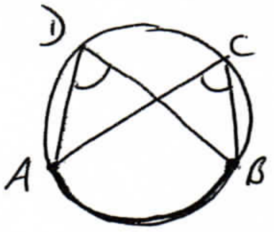
(ii) If PT is ⊥ to OP then PT is a tangent.

(4)



Angle at centre is twice angle at the circle

⇒ (5)

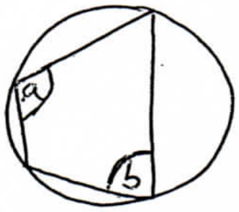


Corollary 1

Angles at the circle on the same arc are equal in measure

ie $|\angle ACB| = |\angle ADB|$

(6)

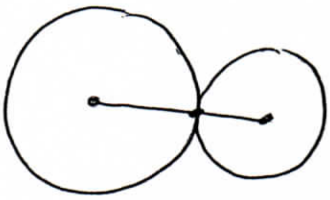


Corollary 2

The sum of opposite angles in a cyclic quadrilateral is 180°

ie $a + b = 180°$.

(7)



Corollary 6

If two circles intersect at one point only, then the two centres and the point of contact are collinear.

F) CONSTRUCTIONS

- Be able to perform all 22 constructions on the course.
- They're in your textbook!
- DON'T FORGET YOUR MATHS CONSTRUCTION KIT.

G) ENLARGEMENTS

① Both size and position of shape changes.

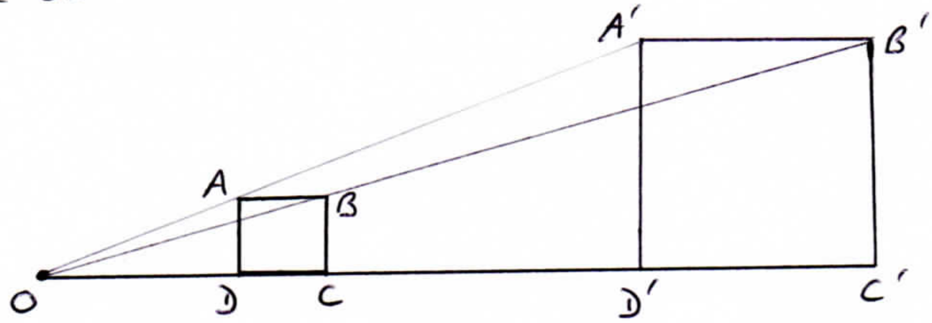
② Enlarge an OBJECT to create an IMAGE.

③ SCALE FACTOR = k
= number by which each length is enlarged to create image

- $k = 2 \Rightarrow$ twice as big
- $k = \frac{1}{2} \Rightarrow$ half as big etc

④ Need to know CENTRE OF ENLARGEMENT, O.

eg.



⑤ In diagram above, $k = 3$.

So (i) $\frac{|A'B'|}{|AB|} = 3$ ie $\boxed{\frac{\text{image length}}{\text{object length}} = k}$

(ii) $\frac{|OA'|}{|OA|} = 3$ ie $\boxed{\text{image point is } k \text{ times as far away from O as object point.}}$

(iii) $\boxed{\frac{\text{Area image}}{\text{Area object}} = k^2}$ ie $\frac{\text{Area } A'B'C'D'}{\text{Area } ABCD} = 9$

(iv) Under an enlargement, the object and image are SIMILAR to each other
(ie angles same, sides parallel, ratio of sides same)

(v) In 3D enlargements :

$$\boxed{\frac{\text{Image Volume}}{\text{Object Volume}} = k^3}$$