End of 5th YR

Summer Revision

Proj (1)

1. 7. From a point p on a plane, inclined at $\tan^{-1}(\frac{1}{2})$ to the horizontal, a particle is projected with speed u at 45° to the plane. The motion takes place in a vertical plane through a line of greatest slope up the plane from p.

Express the velocity $\vec{v}$ and displacement $\vec{r}$ from p of the particle after time t in terms of $\vec{i}$ and $\vec{j}$, where $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the plane, respectively.

Prove that the particle strikes the plane at 90° and that the range on the inclined plane is $\frac{u^2\sqrt{5}}{4g}$.

$$\vec{v} = \left(\frac{u}{\sqrt{2}} - \frac{gt}{\sqrt{5}}\right)\vec{i} + \left(\frac{u}{\sqrt{2}} - \frac{2gt}{\sqrt{5}}\right)\vec{j}$$

$$\vec{r} = \left(\frac{ut}{\sqrt{2}} - \frac{gt^2}{2\sqrt{5}}\right)\vec{i} + \left(\frac{ut}{\sqrt{2}} - \frac{gt^2}{\sqrt{5}}\right)\vec{j}$$

2. 2. A particle is projected from a point O on a plane inclined at 60° to the horizontal with velocity $u = 7\sqrt{3}\,\vec{i} + 4{\cdot}9\,\vec{j}$ metres/second where $\vec{i}$ is a unit vector through O pointing upward along the line of greatest slope in the plane and $\vec{j}$ is a unit vector perpendicular to the plane. Show that after time t seconds the position vector, $\vec{r}$, of the particle relative to O is given by $\vec{r} = \frac{7}{20}[(20\sqrt{3}t - 7\sqrt{3}t^2)\vec{i} + (14t - 7t^2)\vec{j}]$ metres. Prove that the range on the inclined plane is $21\sqrt{3}/5$ metres, and find the velocity of the particle when it strikes the plane.

$$\vec{v} = -\frac{7}{10}\left[u\sqrt{3}\,\vec{i} + 7\vec{j}\right]$$

... is projected up the plane

5. 3. A particle is projected with a speed of 10 m/s at an angle $\alpha°$ to the horizontal up a plane inclined at 30° to the horizontal. If the particle strikes the plane at right angles, show that the time of flight can be represented by the two expressions (1982)

$$\frac{10\cos(\alpha - 30)}{g\sin 30} \quad \text{and} \quad \frac{20\sin(\alpha - 30)}{g\cos 30}$$

$\frac{\sqrt{3}}{2}$

Hence deduce a value for $\tan(\alpha - 30)$.

Calculate the range of the particle along the plane. $\frac{400}{7g}$

8. A plane is inclined at an angle $\tan^{-1}(\frac{1}{2})$ to the horizontal. A particle is projected up the plane with velocity u at an angle θ to the plane. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane parallel to the horizontal.

Express t, the time of flight, in terms of u and θ. time of flight $= \frac{u\sin\theta}{g}\sqrt{5}$

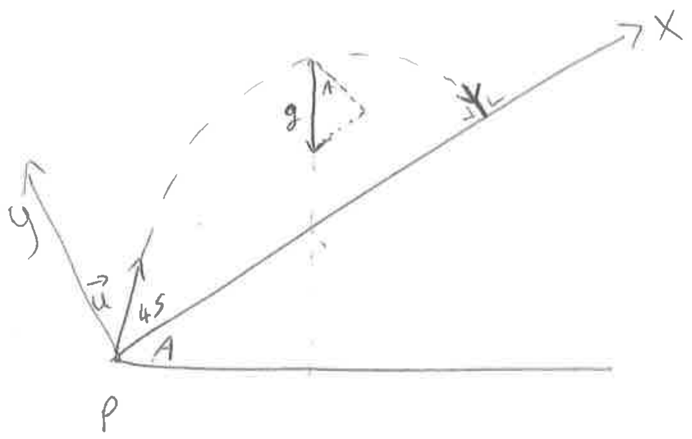
Hence, or otherwise, establish that

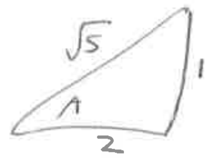
$$\tan\theta = \frac{1}{3}.$$

Calculate the range along the plane. $\frac{u^2\sqrt{5}}{4g}$

Projectiles

1





$$u_x = u\cos 45 = u\cdot 1/\sqrt{2}$$
$$u_y = u\sin 45 = u\cdot 1/\sqrt{2}$$
$$a_x = -g\sin A = -g\cdot 1/\sqrt{5}$$
$$a_y = -g\cos A = -g\cdot 2/\sqrt{5}$$

$v = u + at$

$$V_x = \frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}}t$$
$$V_y = \frac{u}{\sqrt{2}} - \frac{2g}{\sqrt{5}}t$$
$$\vec{v} = \left(\frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}}t\right)\vec{i} + \left(\frac{u}{\sqrt{2}} - \frac{2g}{\sqrt{5}}t\right)\vec{j}$$

$s = ut + \frac{1}{2}at^2$

$$S_x = \frac{u}{\sqrt{2}}t - \frac{g}{2\sqrt{5}}t^2$$
$$S_y = \frac{u}{\sqrt{2}}t - \frac{g}{\sqrt{5}}t^2$$
$$\vec{s} = \left(\frac{ut}{\sqrt{2}} - \frac{gt^2}{2\sqrt{5}}\right)\vec{i} + \left(\frac{ut}{\sqrt{2}} - \frac{gt^2}{\sqrt{5}}\right)\vec{j}$$

Prove: Particle strikes plane at 90° &

Range $= \dfrac{u^2\sqrt{5}}{4g}$

Range ie. S_x when $S_y = 0$

$$\frac{ut}{\sqrt{2}} - \frac{gt^2}{\sqrt{5}} = 0 \qquad t\left(\frac{u}{\sqrt{2}} - \frac{gt}{\sqrt{5}}\right) = 0$$

$$t = 0 \quad \text{or} \quad \boxed{t = \frac{\sqrt{5}u}{\sqrt{2}g}}$$

strike plane at 90° $\Longrightarrow$ no velocity along plane when it hits

$V_x = 0$
(should be!)
lets check:

$V_x = 0$ ~~when~~ at t when $S_y = 0$

$$V = \left(\frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}}t\right)\vec{i} + \left(\frac{u}{\sqrt{2}} - \frac{2g}{\sqrt{5}}t\right)\vec{j}$$

$\downarrow$

sub in $t = \dfrac{\sqrt{5}u}{\sqrt{2}g}$

$$V_x = \frac{u}{\sqrt{2}} - \frac{\cancel{g}}{\cancel{\sqrt{5}}} \cdot \frac{\cancel{\sqrt{5}}u}{\sqrt{2}\cancel{g}} = \frac{u}{\sqrt{2}} - \frac{u}{\sqrt{2}} = 0$$

Proved
q.e.d.

- Find Range: ie find s_x when $t = \dfrac{\sqrt{5}u}{\sqrt{2}g}$

$$S_x = \frac{ut}{\sqrt{2}} - \frac{gt^2}{2\sqrt{5}}$$

$$= \frac{u}{\sqrt{2}}\left(\frac{\sqrt{5}u}{\sqrt{2}g}\right) - \frac{g}{2\sqrt{5}}\left(\frac{\sqrt{5}u}{\sqrt{2}g}\right)^2$$

$$= \frac{\sqrt{5}u^2}{2g} - \frac{\sqrt{5}u^2}{4g}$$

$$= \frac{1}{2}\left(\frac{\sqrt{5}u^2}{g}\right) - \frac{1}{4}\left(\frac{\sqrt{5}u^2}{g}\right)$$

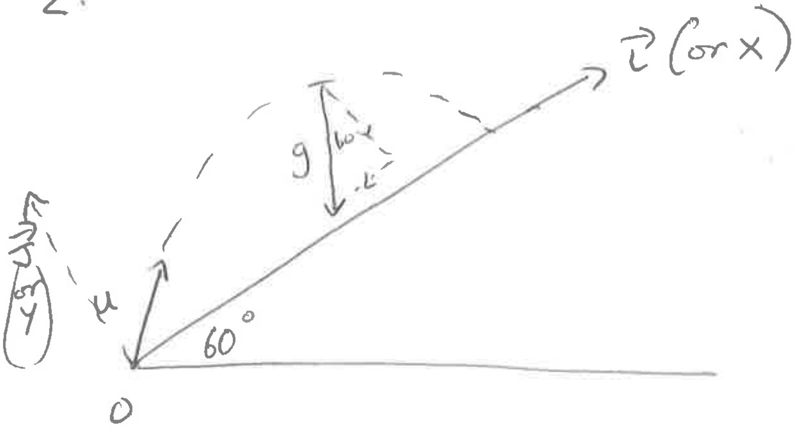
$$= \frac{1}{4}\left(\frac{\sqrt{5}u^2}{g}\right)$$

$$= \boxed{\frac{\sqrt{5}u^2}{4g}}$$

q.e.d.

Projectiles

2.



$u = 7\sqrt{3}\,\vec{i} + 4.9\,\vec{j}$

$u_x = 7\sqrt{3}$

$u_y = 4.9$

$a_x = -g\sin 60 = -g\frac{\sqrt{3}}{2}$

$a_y = -g\cos 60 = -g\cdot\frac{1}{2}$

$v = u + at$

$V_x = 7\sqrt{3} - g\frac{\sqrt{3}}{2}t$

$V_y = 4.9 - \frac{g}{2}t$

$s = ut + \frac{1}{2}at^2$

$S_x = 7\sqrt{3}t - \frac{g\sqrt{3}}{4}t^2$

$S_y = 4.9t - \frac{g}{4}t^2$

$\Rightarrow S = \left[7\sqrt{3}t - \frac{g\sqrt{3}}{4}t^2\right]\vec{i} + \left[4.9t - \frac{g}{4}t^2\right]\vec{j}$ (but $g = 9.8$ or $\frac{98}{10}$)

$= \left[7\sqrt{3}t - \frac{49}{20}\sqrt{3}t^2\right]\vec{i} + \left[\frac{98}{20}t - \frac{49}{20}t^2\right]\vec{j}$

$= \frac{7}{20}\left\{\left[20\sqrt{3}t - 7\sqrt{3}t^2\right]\vec{i} + \left[14t - 7t^2\right]\vec{j}\right\}$

- Prove Range $= \frac{21\sqrt{3}}{5}$.

Range : ie find S_x when $S_y = 0$

$S_y = 0$

$\frac{7}{20}\left[14t - 7t^2\right] = 0$ or $\frac{98t}{20} - \frac{49t^2}{20} = 0$ $(\times 20)$

$t\left[98 - 49t\right] = 0$

$t = 0$ or $\boxed{t = \frac{98}{49} = 2}$

$S_x = \frac{7}{20}\left[20\sqrt{3}t - 7\sqrt{3}t^2\right]$

$= \frac{7}{20}\left[20\sqrt{3}(2) - 7\sqrt{3}(2)^2\right]$

$= \frac{7}{20}\left[40\sqrt{3} - 28\sqrt{3}\right]$

$= \frac{7}{20}\left[12\sqrt{3}\right]$

$= \boxed{\frac{21\sqrt{3}}{5}}$ q.e.d

- V when it strikes $= ?$

ie find V when $t = 2$

$V = V_x \vec{i} + V_y \vec{j}$

$= \left(7\sqrt{3} - \frac{g\sqrt{3}t}{2}\right)\vec{i} + \left(4.9 - \frac{gt}{2}\right)\vec{j}$

$= \left[7\sqrt{3} - \frac{g\sqrt{3}}{2}(2)\right]\vec{i} + \left[4.9 - \frac{g}{2}(2)\right]\vec{j}$

$= \left[7\sqrt{3} - \sqrt{3}g\right]\vec{i} + \left[4.9 - g\right]\vec{j}$

$= \left[7\sqrt{3} - \sqrt{3}\cdot\frac{98}{10}\right]\vec{i} + \left[\frac{49}{10} - \frac{98}{10}\right]\vec{j} =$

$\frac{7}{10}\left[10\sqrt{3} - 14\sqrt{3}\right]\vec{i} + \frac{7}{10}\left[7 - 14\right]\vec{j}$

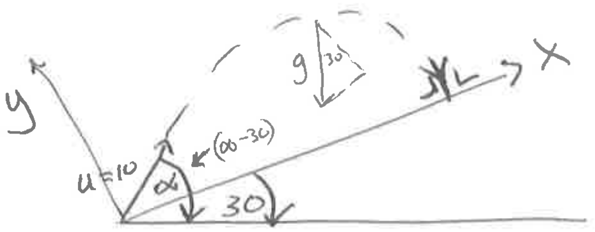
$= \frac{7}{10}\left[-4\sqrt{3}\vec{i} - 7\vec{j}\right]$

$= \boxed{-\frac{7}{10}\left[7\vec{j} + 4\sqrt{3}\vec{i}\right]}$

q.e.d

Projectiles

5.



$u_x = 10\cos(\alpha-30)$

$u_y = 10\sin(\alpha-30)$

$a_x = -g\sin 30 = -g/2$

$a_y = -g\cos 30 = -g\frac{\sqrt{3}}{2}$

$v = u + at$ $\qquad V_x = 10\cos(\alpha-30) - \frac{g}{2}t$

$V_y = 10\sin(\alpha-30) - g\frac{\sqrt{3}}{2}t$

$s = ut + \frac{1}{2}at^2$ $\qquad S_x = 10\cos(\alpha-30)t - \frac{g}{4}t^2$

$S_y = 10\sin(\alpha-30)t - \frac{g\sqrt{3}}{4}t^2$

• Show time of flight (t.o.f) $= \frac{10\cos(\alpha-30)}{g\sin 30}$ & $\frac{20\sin(\alpha-30)}{g\cos 30}$

t.o.f : ie find t when $S_y = 0$ $\qquad \frac{1}{2}g\cos 30\, t^2$

$S_y = 0$: $10\sin(\alpha-30)t - \frac{g\sqrt{3}}{4}t^2 = 0$

$\cancel{t\left(10\sin[\alpha-30] - \frac{g\sqrt{3}}{4}t\right) = 0}$

$\cancel{t = 0 \quad \text{or} \quad t = \frac{40\sin[\alpha-30]}{g\sqrt{3}}}$

Scrap this. go back to previous line + use
(need cos 30 in answer!)

$t\left(10\sin[\alpha-30] - \frac{1}{2}g\cos 30\, t\right) = 0$

$t = 0$ or $\boxed{t = \frac{20\sin(\alpha-30)}{g\cos 30}}$ $\qquad$ q.e.d or

t.o.f : ie find t when $V_x = 0$ — hits plane at 90° angle

$V_x = 0$ $\quad 10\cos(\alpha - 30) - g\sin 30\, t = 0$

$$\boxed{t = \frac{10\cos(\alpha - 30)}{g\sin 30}}$$

q.e.d.

- Deduce value for $\tan(\alpha - 30)$

$t = t \quad \Rightarrow$

$$\frac{20\sin(\alpha - 30)}{g\cos 30} = \frac{10\cos(\alpha - 30)}{g\sin 30}$$

$$\frac{20\cancel{g}\sin 30}{10\cancel{g}\cos 30} = \frac{\cos(\alpha - 30)}{\sin(\alpha - 30)}$$

$$2\tan 30 = \tan(\alpha - 30)$$

$$2 \cdot \frac{1}{\sqrt{3}} = \tan(\alpha - 30)$$

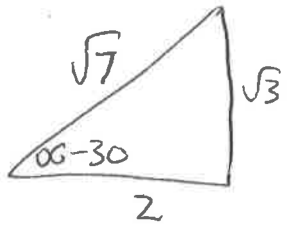
$$\boxed{\frac{2}{\sqrt{3}} = \tan(\alpha - 30)}$$

q.e.d.

50/

$$t = \frac{10\cos(\alpha - 30)}{g\sin 30} \qquad t = \frac{20\sin(\alpha - 30)}{g\cos 30}$$

$$\tan(\alpha - 30) = \frac{\sqrt{3}}{2}$$



Range : find S_x when $S_y = 0$ ie when t = either of above values

$S_x =$

$$10\cos(\alpha - 30)t - \frac{g}{2}\sin 30\, t^2$$

$$= 10 \cdot \frac{2}{\sqrt{7}}\, t - \frac{g}{2} \cdot \frac{1}{2}\, t^2$$

$$= \frac{20}{\sqrt{7}}\, t - \frac{g}{4}\, t^2$$

$$= \frac{20}{\sqrt{7}}\left[\frac{10\cos(\alpha - 30)}{g\sin 30}\right] - \frac{g}{4}\left[\frac{10\cos(\alpha - 30)}{g\sin 30}\right]^2$$

*

$$\frac{20}{\sqrt{7}}\left[\frac{10 \cdot \frac{2}{\sqrt{7}}}{\frac{g}{2}}\right] - \frac{g}{4}\left[\frac{10 \cdot \frac{2}{\sqrt{7}}}{\frac{g}{2}}\right]^2$$

$$\frac{20}{\sqrt{7}}\left[\frac{40}{\sqrt{7}g}\right] - \frac{g}{4}\left[\frac{1600}{7g^2}\right]$$

$$\frac{800}{7g} - \frac{400}{7g} = \frac{400}{7g}$$